Online Appendix: Should platforms be allowed to sell on their own marketplaces?

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A Horizontal differentiation

In this section, we show that the characterizations of the pricing equilibrium in the main text remain valid even if we allow for horizontal product differentiation between S's product and others' products. The main results show that the overall insights on how the dual mode affects the pricing behavior of third-party sellers carry over to this setting.

Consider the following modification to the baseline model in Section 2 which generates horizontal product differentiation in the simplest possible fashion. The consumers' valuations for S's product are $v + \Delta - \mu v_o$, where recall consumers draw the value of their outside option v_o from the distribution G. Here, $\mu \geq 0$ captures the extent of consumer taste heterogeneity for S's innovative product. If $\mu = 0$ then we recover the baseline model. To fix the intuition, it is useful to interpret a consumer's type as his willingness to try out new and innovative products, which is negatively correlated with the value of the consumer's status quo of not buying anything (outside option).

We focus on the case with $\sigma = 0$ and G uniform (with a sufficiently large upperbound $\bar{v}_o$) to facilitate the exposition, but the analysis is easily extendable to the case of $\sigma \neq 0$ and non-uniform and concave G. Given that G is uniform, (2) in the main text becomes

$$\max\{b, \bar{\Delta}\} < v. \tag{A.1}$$

To focus on pricing behavior, we assume that the innovation level Δ is *exogenously fixed* at $\Delta = \Delta^l$ (this assumption is equivalent to assuming $K(\Delta)$ is sufficiently high for all $\Delta > \Delta^l$ in the baseline model in Section 2).

A.1 Pure modes

Seller mode. The new specification for the value of S's product has no effect on the seller mode, so Proposition 2 applies.

Marketplace mode. Following the analysis in the main text, we focus on $\tau \leq b$ so that S sells through the marketplace in equilibrium. With the new specification, consumers buy S's product if and

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only if $v + \Delta + b - \mu v_o - p_i > v_o$ (competition with outside option) and $v + \Delta - \mu v_o - p_i > v - \tau$ (competition with fringe). I.e.,

$$v_o \leq \max\left\{\frac{v + \Delta + b - p_i}{1 + \mu}, \frac{\tau + \Delta - p_i}{\mu}\right\}.$$

The profit function is

$$\pi(p_i) = \left\{ \begin{array}{cc} (p_i - \tau)G\left(\frac{v+\Delta+b-p_i}{1+\mu}\right) & \text{if } p_i \leq \Delta + \tau(1+\mu) - (v+b)\mu \\ (p_i - \tau)G\left(\frac{\tau+\Delta-p_i}{\mu}\right) & \text{if } p_i \geq \Delta + \tau(1+\mu) - (v+b)\mu \end{array} \right\},$$

which is continuous and quasiconcave because each piecewise component is concave with a kink point at which the slope decreases. Moreover, for $p_i < \Delta + \tau(1+\mu) - (v+b)\mu$, notice that

$$\begin{aligned} \frac{d\pi(p_i)}{dp_i} &= \left(\frac{v + \Delta + b + \tau - 2p_i}{1 + \mu}\right) \\ &> \left(\frac{v + \Delta + b + \tau - 2(\Delta + \tau(1+\mu) - (v+b)\mu)}{1 + \mu}\right) \\ &= \left(\frac{(v + b - \tau)(2\mu + 1) - \Delta}{1 + \mu}\right) > 0, \end{aligned}$$

where the last inequality is due to $\tau \leq b$ and (A.1).

Focusing on $p_i \geq \Delta + \tau(1+\mu) - (v+b)\mu$, the first-order condition is

$$p_i = \tau + \frac{\mu G\left(\frac{\tau+\Delta-p_i}{\mu}\right)}{g\left(\frac{\tau+\Delta-p_i}{\mu}\right)} \implies p_i = \tau + \frac{\Delta}{2}$$

given G is uniform. By quasiconcavity of the profit function, we conclude

$$p_i^* = \max\left\{\Delta + \tau(1+\mu) - (v+b)\mu, \tau + \frac{\Delta}{2}\right\}.$$

Note that $\Delta + \tau(1+\mu) - (v+b)\mu \geq \tau + \frac{\Delta}{2}$ if and only if $\mu \leq \frac{\Delta}{2(v+b-\tau)}$.

From the equilibrium of the pricing subgame, we can write down M's profit as

$$\Pi_{\tau \leq b} = \left\{ \begin{array}{cc} \tau\left(G(v+b-\tau) - G\left(\frac{\Delta}{2\mu}\right)\right) + \tau G\left(\frac{\Delta}{2\mu}\right) & \text{if } \mu > \frac{\Delta}{2(v+b-\tau)} \\ \tau G(v+b-\tau) & \text{if } \mu \leq \frac{\Delta}{2(v+b-\tau)} \end{array} \right\}.$$

It is useful to note that whenever μ is large, in the pricing subgame not all consumers buy from S, and a fraction $G(v+b-\tau) - G\left(\frac{\Delta}{2\mu}\right)$ of consumers buy from fringe sellers. Nonetheless, M's profit is the same regardless of μ because the commission is uniform. By the same argument as in the main text, M never sets $\tau > b$ and (A.1) implies $\tau^{mkt} = b$.

To summarize.

Lemma A.1 *(Marketplace mode equilibrium) M sets* $\tau^{mkt} = b$. *S participates and sells exclusively through the marketplace.*

- *If* $\mu \leq \frac{\Delta}{2v}$, *S sets* $p_i^* = \Delta + b - v\mu$ *and sells to all consumers. The profits are* $\Pi = bG(v)$ *and* $\pi = (\Delta - v\mu)G(v)$.
- *If* $\mu > \frac{\Delta}{2v}$, *S sets* $p_i^* = b + \frac{\Delta}{2}$ *and sells to a fraction* $G\left(\frac{\Delta}{2\mu}\right)$ *of consumers. The profits are* $\Pi = bG(v)$ *and* $\pi = \frac{\Delta}{2}G(\frac{\Delta}{2\mu})$.

Notice that if $\mu \to 0$, then we recover Proposition 1 in the main text.

A.2 Dual mode

Consider the construction of the price squeeze equilibrium with $\tau \leq b$, so that we know S necessarily sells through the marketplace in equilibrium. We know that in any equilibrium, M never sets $p_m > \tau$ because consumers would prefer the fringe product on the marketplace over M's offering. So, we focus on $p_m \leq \tau$.

Best response of S**.** Consumers buy from S if and only if $v + \Delta + b - \mu v_o - p_i > v_o$ (competition with outside option) and $v + \Delta - \mu v_o - p_i > v - p_m$ (competition with M). I.e.,

$$v_o \leq \max\left\{\frac{v + \Delta + b - p_i}{1+\mu}, \frac{p_m + \Delta - p_i}{\mu}\right\}.$$

Notice that the first constraint is the relevant one if and only if $p_i < \Delta + p_m(1+\mu) - (v+b)\mu$. Therefore, S's profit function is

$$\pi(p_i; p_m) = \left\{\begin{array}{ll} (p_i - \tau)G\left(\frac{v+\Delta+b-p_i}{1+\mu}\right) & \text{if } p_i \leq \Delta + p_m(1+\mu) - (v+b)\mu \\ (p_i - \tau)G\left(\frac{p_m+\Delta-p_i}{\mu}\right) & \text{if } p_i \geq \Delta + p_m(1+\mu) - (v+b)\mu \end{array}\right\}. \tag{A.2}$$

By the same analysis as in the marketplace mode, we know S sets $p_i \geq \Delta + p_m(1+\mu) - (v+b)\mu$ for all $p_m \leq \tau$. Focusing on the second row of (A.2), the first-order condition is

$$p_i = \tau + \frac{\mu G\left(\frac{p_m+\Delta-p_i}{\mu}\right)}{g\left(\frac{p_m+\Delta-p_i}{\mu}\right)} \implies p_i = \frac{p_m + \Delta + \tau}{2}$$

given G is uniform. So, S's best response function is

$$p_i^{BR}(p_m) = \max\left\{\frac{p_m + \Delta + \tau}{2}, \Delta + p_m(1+\mu) - (v+b)\mu\right\}.$$

Best response of M. Consumers buy from M if and only if $p_m \leq \tau$ (competition with fringe), $v + b - p_m > v_o$ (competition with the outside option) and $v - p_m > v + \Delta - \mu v_o - p_i$ (competition with S). Combining the latter two conditions,

$$\frac{p_m + \Delta - p_i}{\mu} < v_o < v + b - p_m.$$

This range is non-empty if and only if $p_i \geq \Delta + p_m(1+\mu) - (v+b)\mu$, or rearranging, $p_m < \frac{(v+b)\mu + p_i - \Delta}{1+\mu}$. Then M's profit function is:

$$\Pi(p_m; p_i) = \begin{cases} p_m \left(G\left(v + b - p_m\right) - G\left(\frac{p_m + \Delta - p_i}{\mu}\right)\right) + \tau G\left(\frac{p_m+\Delta-p_i}{\mu}\right) & \text{if } p_m \leq \frac{(v+b)\mu+p_i-\Delta}{1+\mu} \\ \tau G\left(\frac{v+\Delta+b-p_i}{1+\mu}\right) & \text{if } p_m \geq \frac{(v+b)\mu+p_i-\Delta}{1+\mu} \end{cases},$$

which is clearly quasiconcave. Focusing on the first row, the first-order condition is

$$G\left(v + b - p_m\right) - G\left(\frac{p_m + \Delta - p_i}{\mu}\right) - p_m g\left(v + b - p_m\right) - \frac{p_m - \tau}{\mu} g\left(\frac{p_m + \Delta - p_i}{\mu}\right) = 0.$$

Applying uniformity of G, the solution to the first-order condition is

$$p_m = \frac{(v+b)\,\mu + \tau + p_i - \Delta}{2\,(1+\mu)}.$$

Hence, we can summarize M's best response function as

$$p_m^{BR}(p_i) = \min\left\{\frac{(v+b)\,\mu + \tau + p_i - \Delta}{2\,(1+\mu)}, \frac{(v+b)\,\mu + p_i - \Delta}{1+\mu}, \tau\right\}.$$

Equilibrium. Combining the best response functions leads to the following characterization of the equilibrium in the pricing subgame:

Lemma A.2 *Suppose $\tau \leq b$. In the pricing subgame:*

- *If $\mu \leq \frac{\Delta}{2v+2b-\frac{\tau}{1+\mu}}$, any price profile satisfying $p_i^* = \Delta + p_m^*(1+\mu) - (v+b)\mu$ and*

$$p_m^* \in \left[\max\left\{\frac{2(v+b)\mu - \Delta + \tau}{1+2\mu}, 0\right\}, \frac{\tau}{1+\mu}\right] \tag{A.3}$$

  is an equilibrium. The equilibrium profits are $\Pi = \tau G\left(v + b - p_m^*\right)$ and $\pi = (\Delta + p_m^*(1+\mu) - (v+b)\mu - \tau) G\left(v + b - p_m^*\right)$.

- *If $\mu \geq \frac{\Delta}{2v+2b-\frac{\tau}{1+\mu}}$, in equilibrium M sets $p_m^* = \min\left\{\frac{2(v+b)\mu+3\tau-\Delta}{3+4\mu}, \tau\right\}$ and S sets $p_i^* = \frac{p_m^*+\Delta+\tau}{2}$.*

The equilibrium profits are

$$\Pi = p_m^* \left(G\left(v+b-p_m^*\right) - G\left(\frac{p_m^*+\Delta-p_i}{\mu}\right)\right) + \tau G\left(\frac{p_m^*+\Delta-p_i^*}{\mu}\right)$$

and $\pi = (p_i^* - \tau)G\left(\frac{p_m^*+\Delta-p_i^*}{\mu}\right)$.

Proof. The first equilibrium is the combination of the following conditions:

- Price above marginal cost: $p_m^* \geq 0$.
- S's best response: $p_i^{BR}(p_m^*) = \Delta + p_m^*(1+\mu) - (v+b)\mu \geq \frac{p_m^*+\Delta+\tau}{2}$, which is equivalent to $p_m^* \geq \frac{2(v+b)\mu-\Delta+\tau}{1+2\mu}$. Notice this condition implies $p_i^* \geq \tau$ because

$$p_i^* - \tau \geq \frac{p_m+\Delta+\tau}{2} - \tau \geq \frac{\frac{2(v+b)\mu-\Delta+\tau}{1+2\mu}+\Delta+\tau}{2} - \tau > 0,$$

  where the final inequality uses $v+b > \tau - \Delta$.
- M's best response: $p_m^{BR}(p_i^*) = \frac{(v+b)\mu+p_i^*-\Delta}{1+\mu} \leq \frac{(v+b)\mu+\tau+p_i^*-\Delta}{2(1+\mu)}$, which is equivalent to $\tau \geq (v+b)\,\mu + p_i^* - \Delta$. Substituting for $p_i^* = \Delta + p_m^*(1+\mu) - (v+b)\mu$, this is equivalent to $\tau \geq p_m^*(1+\mu)$.
- The set (A.3) is non-empty if and only if $\frac{2(v+b)\mu-\Delta+\tau}{1+2\mu} \leq \frac{\tau}{1+\mu}$, which is equivalent to $\mu \leq \frac{\Delta}{2v+2b-\frac{\tau}{1+\mu}}$.

For the second equilibrium, the condition $\mu > \frac{\Delta}{2v+2b-\frac{\tau}{1+\mu}}$ implies that there can be no equilibrium with $p_i^* = \Delta + p_m^*(1+\mu) - (v+b)\mu$. This is because any $p_m^* > \frac{\tau}{1+\mu}$ implies M would deviate from such an equilibrium, whereas $p_m^* \leq \frac{\tau}{1+\mu} < \frac{2(v+b)\mu-\Delta+\tau}{1+2\mu}$ implies S would deviate from such an equilibrium (the second inequality is due to $\mu > \frac{\Delta}{2v+2b-\frac{\tau}{1+\mu}}$). After ruling out this possibility, the best response functions imply that, in equilibrium, S sets $p_i^* = \frac{p_m^*+\Delta+\tau}{2}$ and M sets

$$p_m^* = \min\left\{\frac{(v+b)\,\mu+\tau+p_i^*-\Delta}{2\,(1+\mu)}, \tau\right\}.$$

Solving the simultaneous equations, we have $p_i^* = \frac{p_m^*+\Delta+\tau}{2}$ and $p_m^* = \min\left\{\frac{2(v+b)\mu+3\tau-\Delta}{3+4\mu}, \tau\right\}$. ■

Notice that the set of equilibria in (A.3) is reminiscent of the set of equilibria in (5) in the baseline model. By the same equilibrium selection rule, we select $p_m^* = \max\left\{\frac{2(v+b)\mu-\Delta+\tau}{1+2\mu}, 0\right\}$ in (A.3).

Next, by the same argument as in the main text, M never sets $\tau > b$ and (A.1) implies $\tau = b$ is optimal. To summarize:

Lemma A.3 *(Dual mode equilibrium) M sets $\tau^{dual} = b$. S participates and sells exclusively through the marketplace.*

- *If $\mu \leq \frac{\Delta}{2v+2b-\frac{b}{1+\mu}}$, the prices are $p_i^* = \Delta + p_m^*(1+\mu) - (v+b)\mu$ and $p_m^* = \max\left\{\frac{2v\mu-\Delta}{1+2\mu} + b, 0\right\}$. M sells to no consumers in equilibrium.*

- *If* $\mu \geq \frac{\Delta}{2v+2b-\frac{b}{1+\mu}}$*, the prices are* $p_i^* = \frac{p_m^*+\Delta+b}{2}$ *and* $p_m^* = \min\left\{\frac{2(v+b)\mu+3b-\Delta}{3+4\mu}, b\right\}$*. M sells to some consumers in equilibrium.*

The case $\mu \leq \frac{\Delta}{2v+2b-\frac{b}{1+\mu}}$ is analogous to the equilibrium in Proposition 3 in the main text (the case where $b > \bar{\tau}$, given the assumption of Δ being exogenous and $\sigma = 0$). In equilibrium, M sets $p_m^* < \tau$ and imposes a "price squeeze" on S's inside price. The squeeze becomes weaker when the extent of horizontal differentiation increases, i.e., $p_m^* = \max\left\{\frac{2v\mu-\Delta}{1+2\mu} + b, 0\right\} \leq b$ is increasing in μ. Notice that if $\mu \to 0$, we recover Proposition 3 exactly.

A.3 Banning dual mode

Recall that we have assumed $\sigma = 0$, which implies that M is always indifferent between switching to the pure marketplace mode and the pure seller mode after the dual mode is banned. We consider both possibilities in the analysis below.

If $\mu < \frac{\Delta}{2v+2b-\frac{b}{1+\mu}}$, the ban has the following implications

M's choice of mode	Π	π	CS	W
Seller	↓	↓	↓	↓
Marketplace	↓	↑	↓	↓

"." = not changing; " ↑ " = increasing; " ↓ " = decreasing.

The results are similar to Proposition 4 (given that we have fixed the level of innovation).

If $\mu > \frac{\Delta}{2v+2b-\frac{b}{1+\mu}}$, M is indifferent between all three modes. The ban has the following implications

M's choice of mode	Π	π	CS	W
Seller	.	↓	.	↓
Marketplace	.	.	.	.

"." = not changing; " ↑ " = increasing; " ↓ " = decreasing.

The equilibrium demand is the same across all three modes, so that the ban does not affect consumer surplus. The switch to the seller mode decreases S's profit and welfare because S no longer sells to any consumer.

The switch to the marketplace mode has no effect because the market outcome is the same as in dual mode. It should be emphasized that this result is partly driven by the assumption of $\sigma = 0$. If $\sigma > 0$, then in the dual mode equilibrium M optimally sets $p_m^* \in (\tau, \tau + \sigma]$, so that it strictly prefers the dual mode. In that case, the ban on the dual mode (resulting in a switch to the marketplace mode) decreases welfare because M's product advantage σ is lost. The ban would also decrease consumer surplus if $p_m^* < \tau + \sigma$, which is true when σ is sufficiently large so that M's price is not bound by the competition with fringe sellers.

B Baseline model with percentage fees

In this section, we show that the key insights of the baseline model remain valid when the platform charges percentage fees. Suppose that for each unit of sales revenue on the marketplace, a seller receives its share $1 - r$ whereas the platform keeps the remaining share $r \in [0, 1]$. With percentage fees, the level of marginal costs of the products matters. Thus, rather than normalizing at zero, the marginal costs of all products are normalized to $c > 0$, where $c < \min\{v, b + \sigma, \Delta^l\}$. Given Bertrand competition, fringe sellers always price at effective marginal cost, i.e. c if selling directly and $\frac{c}{1-r}$ if selling on a marketplace.

For simplicity, we assume that the innovation level Δ is exogenously fixed at $\Delta = \Delta^l$. We focus on the interesting case where $0 \leq \sigma \leq \Delta^l$. The following assumption is analogous to assumption (2):

$$\max\left\{b + \sigma, \Delta^l\right\} < \frac{G(v - c)}{g(v - c)} \tag{B.1}$$

All other specifications remain the same as in Section 2.

□ **Marketplace mode.** Suppose $\frac{c}{1-r} \leq b + c$ so that consumers prefer the fringe product on the marketplace over the fringe product in the direct channel. In equilibrium, S adopts exactly one of the following strategies:

- Set $p_o > p_i - b$ (so that any consumer buying from S does so through M) and p_i solves

$$\max_{p_i \leq \frac{c}{1-r} + \Delta^l} ((1 - r)p_i - c)G(v + b + \Delta^l - p_i).$$

  Assumption (B.1) implies $p_i^* = \frac{c}{1-r} + \Delta^l$ is optimal, with S earning a margin of $\Delta^l(1 - r)$.

- Set $p_i > p_o + b$ (so that any consumer buying from S does so directly) and p_o solves

$$\max_{p_o \leq \frac{c}{1-r} + \Delta^l - b} p_o G(v + \Delta^l - p_o)).$$

  Assumption (B.1) implies $p_o^* = \frac{c}{1-r} + \Delta^l - b$ is optimal, and S's margin is exactly $p_o^* - c$.

Comparing S's margin, note that S optimally chooses the first strategy if r is not too large such that

$$\frac{c}{1 - r} \leq b + c - \Delta^l r \tag{B.2}$$

holds, and chooses the second strategy otherwise. This reflects the showrooming constraint, as in the baseline model. Constraint (B.2) implies that M's commission margin $rp_i^* = r\left(\frac{c}{1-r}+\Delta^l\right) \leq b$, i.e., M's commission margin is never higher than the convenience benefit it provides. The logic of the showrooming constraint implies M earns zero profit whenever it sets r that violates (B.2). Thus, M chooses r to maximize

$$\Pi = r\left(\frac{c}{1-r}+\Delta^l\right) G\left(v+b-\frac{c}{1-r}\right),$$

subject to (B.2). Assumption (B.1) implies (B.2) binds. To see this, note that the profit derivative $\frac{d\Pi}{dr}$ has the same sign as

$$\begin{aligned}
& \frac{G\left(v+b-\frac{c}{1-r}\right)}{g\left(v+b-\frac{c}{1-r}\right)} - \frac{\frac{rc}{(1-r)^2}}{\frac{c}{1-r}+\Delta^l+\frac{rc}{(1-r)^2}}\left(\frac{c}{1-r}+\Delta^l\right) \\
= & \frac{G\left(v+b-\frac{c}{1-r}\right)}{g\left(v+b-\frac{c}{1-r}\right)} - \frac{\frac{c}{(1-r)^2}}{\Delta^l+\frac{c}{(1-r)^2}} r\left(\frac{c}{1-r}+\Delta^l\right) \\
\geq & \frac{G\left(v-c+\Delta^l r\right)}{g\left(v-c+\Delta^l r\right)} - b > 0,
\end{aligned}$$

where the second and third inequalities are due to (B.2) and (B.1) respectively.

Thus, M optimally sets $r^{mkt} \in (0,1)$ that is defined as the unique solution of

$$\frac{c}{1-r^{mkt}} + \Delta^l = \frac{b}{r^{mkt}}. \tag{B.3}$$

Equilibrium profits are $\Pi^{mkt} = bG\left(v-c+r^{mkt}\Delta^l\right)$ and $\pi^{mkt} = \Delta^l\left(1-r^{mkt}\right)G\left(v-c+r^{mkt}\Delta^l\right)$.

☐ **Seller mode.** M chooses p_m to maximize

$$\max_{p_m \leq b+\sigma+c} (p_m - c)G(v+b+\sigma-p_m).$$

Assumption (B.1) implies that in equilibrium M sets $p_m^* = b+\sigma+c$ and sells to all consumers, whereas S sells to no one. Equilibrium profits are $\Pi^{sell} = (b+\sigma)G(v-c)$ and $\pi^{sell} = 0$.

As opposed to the baseline model with constant per-transaction fees, notice that $\sigma > 0$ does not necessarily imply $\Pi^{sell} > \Pi^{mkt}$ given the greater transaction volume in the pure marketplace mode. The key intuition is that, relative to per-transaction fees, percentage fees allow M to achieve the same commission margin b but induce a lower price charged by S, consistent with the standard insight in models of vertical relations.

☐ **Dual mode.** Following the logic in Section 3, as long as r is not too large, we have the following equilibrium in the pricing subgame:

- *(Price squeeze equilibrium)* If $\Delta^l > \sigma$, all consumers buy from S through the marketplace. Any price profile satisfying $p_i^* = p_m^* + \Delta^l - \sigma$, $p_o^* \geq p_i^* - b$, and

$$p_m^* \in \left[\max\left\{\frac{c}{1-r} - \Delta^l + \sigma, c\right\}, \min\left\{\frac{c}{1-r} + \min\left\{\sigma, \frac{r(\Delta^l - \sigma)}{1-r}\right\}, \frac{b}{r} - \Delta^l + \sigma\right\}\right] \tag{B.4}$$

is an equilibrium.

Some comments on the construction of the upperbound in (B.4) are in order. If $p_m^* > \frac{c}{1-r} + \sigma$, consumers prefer the fringe product on the marketplace over M's offering. If $p_m^* > \frac{c}{1-r} + \frac{r(\Delta^l - \sigma)}{1-r}$, M has an incentive to undercut because it implies $p_m^* - c > p_i^* r$. Finally, $p_m^* > \frac{b}{r} - \Delta^l + \sigma$ violates the showrooming constraint because S's margin from selling directly would be greater than S's equilibrium margin, that is,

$$p_m^* + \Delta^l - \sigma - b - c > (1-r)(p_m^* + \Delta^l - \sigma) - c.$$

Thus, an interesting distinction relative to the baseline model with constant per-transaction fees is that the equilibrium level of p_m^* affects S's incentive to induce showrooming. Rearranging, the showrooming constraint implies M's commission margin is again bounded above by the convenience benefit it provides, i.e.

$$rp_i^* = r\left(p_m^* + \Delta^l - \sigma\right) \leq b \tag{B.5}$$

We select the lowest price $p_m^* = \max\left\{\frac{c}{1-r} - \Delta^l + \sigma, c\right\}$ in (B.4). Notice that (B.4) is non-empty if and only if

$$\frac{b}{r} - \Delta^l + \sigma \geq \max\left\{\frac{c}{1-r} - \Delta^l + \sigma, c\right\}, \tag{B.6}$$

which represents the showrooming constraint on the commission rate r as discussed above. Then M chooses r to maximize

$$\Pi = r\left(p_m^* + \Delta^l - \sigma\right) G\left(v + b + \sigma - p_m^*\right),$$

subject to $p_m^* = \max\left\{\frac{c}{1-r} - \Delta^l + \sigma, c\right\}$ and (B.6). Using the same argument as in the pure marketplace mode, it is easy to check that (B.1) implies (B.6) binds. Given the maximum operator, there are two ways in which (B.6) can bind. Define $r_1 \in (0,1)$ as the unique solution to

$$\frac{c}{1-r_1} = \frac{b}{r_1},$$

then:

- Case 1: if $b > \Delta^l - \sigma$, then (B.6) binds when $p_m^* = \frac{c}{1-r_1} - \Delta^l + \sigma > c$. In equilibrium M sets $r^{dual} = r_1$. Comparing the definition of r_1 with (B.3), we have $r^{dual} \geq r^{mkt}$. M's profit is $\Pi^{dual} = bG\left(v + \Delta^l + b - \frac{c}{1-r_1}\right) = bG\left(v + \Delta^l - c\right)$.

- Case 2: if $b \leq \Delta^l - \sigma$, then (B.6) binds when $p_m^* = c \geq \frac{c}{1-r_1} - \Delta^l + \sigma$. In equilibrium M sets $r^{dual} = \frac{b}{c+\Delta^l-\sigma} \leq r_1$. Rearranging this as $c + \Delta^l - \sigma = \frac{b}{r^{dual}}$ and comparing with (B.3), we have $r^{dual} \geq r^{mkt}$. M's profit is $\Pi^{dual} = bG(v - c + b + \sigma)$.

Combining both cases, M's profit is

$$\Pi^{dual} = bG\left(v - c + \min\{\Delta^l, b + \sigma\}\right).$$

We note that the equilibrium demand and profit are the same as in the dual mode of the baseline model with per-transaction fees (if we had allowed $c > 0$ and imposed $\Delta = \Delta^l$ in the baseline model). At first glance, with percentage fees one might expect that M may have a weaker incentive to induce a price squeeze because doing so decreases M's commission margin for each given r. However, the price squeeze also relaxes the showrooming constraint (B.5), which allows M to charge a higher commission rate r and still keeping S onboard. Given that r is endogenous and that M's margin is at most b by (B.5), it follows that lowering the price as much as possible is optimal for M. Nonetheless, given that percentage fees generally lead to a lower price by S (relative to per-transaction fees), we can interpret percentage fees as partially substituting for the role of the price squeeze in expanding demand in dual mode.

□ **Discussion.** The analysis above implies that the use of percentage fees (relative to per-transaction fees) does not affect the equilibrium demand and M's profit in the pure seller mode and the dual mode (if we allow $c > 0$ and impose $\Delta = \Delta^l$ in both cases), but raises those in the pure marketplace mode. This shifts the tradeoff (in terms of consumer surplus and welfare) towards the marketplace mode from both the seller mode and the dual mode.

From the dual mode profit expression, note that

$$\begin{aligned} bG(v - c + b + \sigma) &\geq bG(v - c + b) \\ &> bG\left(v - c + r^{mkt}\Delta^l\right) = \Pi^{mkt} \end{aligned}$$

(by the definition of r^{mkt}) and $bG\left(v - c + \Delta^l\right) > \Pi^{mkt}$. Thus $\Pi^{dual} > \Pi^{mkt}$, reflecting that the equilibrium demand is higher in the dual mode than in the pure marketplace mode. Moreover, recall that the pure marketplace mode has a higher equilibrium demand than the pure seller mode. Thus, we conclude that, relative to the pure modes, dual mode still has an advantage of expanding the transaction volume by disciplining S's price even when M charges percentage fees (given $\Delta = \Delta^l$ is fixed). As such, the key trade-offs and implications of banning the dual mode should be similar to Proposition 4 (holding the innovation level fixed).

C Sequential pricing model

In this section, we verify the claim that the equilibrium selection rule in our model would also be implied if M had commitment power in its pricing. Consider the following amendment to the timing: In the pricing subgame, M sets its price first before all third party sellers, including S. Notice that this timing assumption affects only the dual mode. We rule out negative prices throughout this section.

Suppose M chooses the dual mode, and consider first $\tau \in (-\sigma, b]$. To analyze the pricing subgame, we first note that M never sets $p_m > \tau + \sigma > 0$ in any equilibrium of the pricing subgame because then consumers would prefer the fringe product on the marketplace over M's offering. Such prices are dominated by $p_m \leq \tau + \sigma$.

Pricing by S. By backward induction, consider S's post-participation pricing decision in stage 3. Provided that $p_o^* > p_i^* - b$, any consumer buying from S must do so through M. Then, consumers prefer S's product over M's offering if and only if $\Delta - p_i \geq \sigma - p_m$. Hence, if $p_m < \tau + \sigma - \Delta$, then S cannot profitably make any sales without pricing below its effective marginal cost τ, and so it chooses $p_i = \tau$ and earns zero profit. If $p_m \in [\tau + \sigma - \Delta, \tau + \sigma]$, then S's pricing problem is

$$\max_{p_i \leq p_m - \sigma + \Delta} (p_i - \tau) G(v + b + \Delta - p_i).$$

Given that $p_m \leq \tau + \sigma$ so that $p_m - \sigma + \Delta \leq \tau + \Delta$, (2) implies that S's pricing constraint must bind, so it sets $p_i^* = p_m - \sigma + \Delta$. Moreover, given that $\tau \leq b$, there is no incentive for S to deviate by inducing consumers to switch to buy from the direct channel.

Pricing by M. It has two possible pricing strategies (anticipating S's responses):

1. *Limit pricing*: This corresponds to semi-seller mode equilibrium in the simultaneous pricing model in Section 3. If M sets a low price at $p_m < \max\{\tau + \sigma - \Delta, 0\}$, it prevents S from making any sales. M's optimal price and profit in this case are $p_m^* = \tau + \sigma - \Delta$ and $\max\{\tau + \sigma - \Delta, 0\} G(v + \Delta + b - \tau)$ respectively.

2. *Price squeeze*: This corresponds to price squeeze equilibrium in the simultaneous pricing model the baseline dual mode in Section 3. If M sets $p_m \geq \max\{\tau + \sigma - \Delta, 0\}$, then S will sell to all consumers in equilibrium at $p_i^* = p_m - \sigma + \Delta$ and M generates commission revenue τ. M's profit is

$$\max_{p_m > \max\{\tau + \sigma - \Delta, 0\}} \tau G(v + b + \sigma - p_m).$$

By setting a relatively low price at $p_m = \max\{\tau + \sigma - \Delta, 0\}$, M squeezes S's sales margin and induces S to set $p_i^* = \max\{\tau + \sigma - \Delta, 0\} - \sigma + \Delta = \tau$. M's profit is

$$\Pi = \tau G(v + \sigma + b - \max\{\tau - \Delta + \sigma, 0\}).$$

It is easy to verify that M optimally chooses limit pricing if $\sigma \geq \Delta$, and chooses price squeeze otherwise. The equilibrium characterization of the pricing subgame is thus exactly the same as that in Section 3 (given the equilibrium selection rule in the simultaneous pricing model). Thus, the remaining analysis in the baseline dual mode in Section 3 applies.

D Constrained imitation and commitment

In this proof, we focus on $\alpha > 0$ because $\alpha = 0$ corresponds to the dual mode in Section 4. With endogenous probability $\alpha > 0$ platform M is unable to engage in product imitation. Recall $\bar{\Delta}_\alpha$ is the solution to the first-order condition

$$\alpha G(v + b + \sigma) = K'(\bar{\Delta}_\alpha),$$

and

$$\bar{\tau}_\alpha \in (\Delta^l - \sigma, \bar{\Delta}_\alpha - \sigma)$$

is the unique solution of $\alpha(\bar{\Delta}_\alpha - \sigma - \bar{\tau}_\alpha)G(v + b + \sigma) - K(\bar{\Delta}_\alpha) = 0$, which are the counterparts of (1) and (7) in the baseline model. To focus on the interesting case, we assume $\bar{\Delta}_\alpha > \sigma$.

The equilibrium in the stage-3 pricing subgames (with and without imitation) are described in the proof of Proposition 5. Consider the innovation and imitation decisions in stage 2. The analysis below is largely similar to that in the proof of Proposition 8.

Suppose τ is such that $p_i^{show} - \Delta < -\sigma$ for all Δ. With probability α, imitation does not occur. Notice that the constraint $p_i^* \leq p_i^{show}$ in (20) always binds in the price squeeze equilibrium (without imitation) whenever it arises. Hence, regardless of which type of equilibrium applies in the no-imitation pricing subgame, M's profit is always $\Pi = \Pi_{no\text{-}imi}^{exploit}$. With probability $1 - \alpha$, imitation occurs so that S's profit is necessarily zero. Taking into account both possibilities,

$$\pi(\Delta) = \alpha \max\left\{p_i^{show}(\Delta) - \tau, 0\right\} G(v + b + \Delta - p_i^{show}(\Delta)) - K(\Delta).$$

Define $\tilde{\Delta}_\alpha$ as the solution to $\alpha G(v+b+\Delta-p_i^{show}(\Delta)) = K'(\tilde{\Delta}_\alpha)$, where recall $\Delta-p_i^{show}(\Delta)$ is independent of Δ. Let $\tilde{\tau}_\alpha$ be the unique solution of $\alpha(p_i^{show}(\Delta) - \tilde{\tau}_\alpha)G(v + b + \Delta - p_i^{show}(\Delta)) - K(\tilde{\Delta}_\alpha) = 0$, so that S optimally chooses $\tilde{\Delta}_\alpha$ if $\tau \leq \tilde{\tau}_\alpha$ and chooses Δ^l if $\tau > \tilde{\tau}_\alpha$. Then, taking into account probabilistic product imitation, M's profit is

$$\begin{aligned} \Pi_{\tau \leq \tilde{\tau}_\alpha} &= \alpha \Pi_{no\text{-}imi}^{exploit} + (1 - \alpha)(\min\{\tau, b\} + \tilde{\Delta}_\alpha)G(v + b - \min\{\tau, b\}) \\ \Pi_{\tau > \tilde{\tau}_\alpha} &= \Pi_{no\text{-}imi}^{exploit}. \end{aligned}$$

Suppose τ is such that $p_i^{show} - \Delta \geq -\sigma$. Then the constraint $p_i^* \leq p_i^{show}$ in (20) never binds. To make the dependency of p_i^{show} on Δ explicit, we write $p_i^{show} = p_i^{show}(\Delta)$, so that

$$\pi(\Delta) = \begin{cases} -K(\Delta) & \text{if } p_i^{show}(\Delta) < \tau \\ \alpha \max\{\Delta - \sigma - \tau, 0\} G(v + b + \sigma) - K(\Delta) & \text{if } p_i^{show}(\Delta) \geq \tau \end{cases}. \tag{D.1}$$

The same argument as used in the proof of Proposition 8 implies that S optimally chooses $\bar{\Delta}_\alpha > \tau + \sigma$ if $\tau \leq \bar{\tau}_\alpha$ and chooses Δ^l if $\tau > \bar{\tau}_\alpha$. Then, taking into account probabilistic product imitation, M's profit is

$$\begin{aligned} \Pi_{\tau \leq \bar{\tau}_\alpha} &= \alpha \tau G(v + \sigma + b) + (1 - \alpha)(\min\{\tau, b\} + \bar{\Delta}_\alpha) G(v + b - \min\{\tau, b\}) \\ \Pi_{\tau > \bar{\tau}_\alpha} &= \begin{cases} \alpha \max\left\{\tau G(v + \Delta^l + b - \tau), \Pi^{exploit}_{no\text{-}imi}\right\} + (1 - \alpha)(b + \max\{\sigma, \Delta^l\}) G(v) & \text{if } \tau \leq b + \Delta^l \text{ and } \sigma < \Delta^l \\ \Pi^{exploit}_{no\text{-}imi} & \text{if } \tau > b + \Delta^l \text{ or } \sigma \geq \Delta^l \end{cases}. \end{aligned}$$

Denote

$$\Pi^*_\alpha = \alpha \bar{\tau}_\alpha G(v + \sigma + b) + (1 - \alpha)(\min\{\bar{\tau}_\alpha, b\} + \bar{\Delta}_\alpha) G(v + b - \min\{\bar{\tau}_\alpha, b\}),$$

which is M's profit if it sets $\bar{\tau}_\alpha$ and if S responds by choosing innovation $\bar{\Delta}_\alpha$.

Lemma D.1 *(Dual mode equilibrium constrained imitation) Suppose with exogenous probability $\alpha > 0$, platform M is unable to engage in product imitation.*

- *If $b + \max\{\sigma, \Delta^l\} \leq \bar{\tau}_\alpha$ or*

$$\Pi^*_\alpha \geq (b + \max\{\sigma, \Delta^l\}) G(v), \tag{D.2}$$

  then M sets $\tau^{dual} = \bar{\tau}_\alpha$ and S participates and sets $\bar{\Delta}_\alpha$. With probability α, S sells to all consumers through the marketplace and the prices are $p_i^* = \bar{\Delta}_\alpha - \sigma$, $p_o^* \geq p_i^* - b$ and $p_m^* = 0$. With probability $1 - \alpha$, M sells to all consumers and the prices are $p_m^* = \min\{\bar{\tau}_\alpha, b\} + \bar{\Delta}_\alpha$, $p_i^* = \bar{\tau}_\alpha$, and $p_o^* \geq p_i^* - b$.

- *If $b + \max\{\sigma, \Delta^l\} > \bar{\tau}_\alpha$ and (D.2) does not hold, M sets $\tau^{dual} = b + \Delta^l$ and S sets Δ^l. If $\Delta^l > \sigma$, S sells to all consumers exclusively through the marketplace and the prices are $p_i^* = \tau^{dual}$, $p_o^* \geq p_i^* - b$, and $p_m^* = b - \Delta^l + \sigma$. If $\Delta^l \leq \sigma$, M sells to all consumers and the prices are $p_m^* = b + \sigma$, $p_i^* = \tau^{dual}$, and $p_o^* \geq p_i^* - b$.*

Proof. The proof of Lemma 3 applies after taking into account the new profit expressions. It is useful to note that $\tilde{\Delta}_\alpha < \bar{\Delta}_\alpha$ and $\tilde{\tau}_\alpha < \bar{\tau}_\alpha$ (given $p_i^{show} - \Delta < -\sigma$ in the definition of $\tilde{\tau}_\alpha$). This property allows us to establish that any profit that M earns from setting τ that induces $p_i^{show} - \Delta < -\sigma$ must be lower than the profit from setting either $\tau^{dual} = \bar{\tau}_\alpha$ or $\tau^{dual} = b + \Delta^l$. ■

Implications of banning dual mode. The equilibrium characterization in this lemma is different from Proposition 5 (perfect and unconstrained imitation) only if M sets $\tau^{dual} = \bar{\tau}_\alpha$. Thus, if $b + \Delta^l > \bar{\tau}_\alpha$

holds and (D.2) does not hold, Proposition 6 applies. Otherwise, if $b + \Delta^l \leq \bar{\tau}_\alpha$ holds or (D.2) holds, we have

	M's choice of mode	Π	π	CS	Δ	W
if $\sigma > 0$	Seller	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
if $\sigma \leq 0$	Marketplace	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$

"." = not changing; " $\uparrow$ " = increasing; " $\downarrow$ " = decreasing.

To sum up, having constrained imitation moves the result closer to Proposition (4).

Endogenous α. In the dual mode equilibrium with constrained imitation, M's profit is

$$\Pi_\alpha^{dual} \equiv \max\{\Pi_\alpha^*, (b + \max\{\sigma, \Delta^l\})G(v)\}.$$

We claim that $\Pi^*_{\alpha\in(0,1)} > \Pi^*_{\alpha=1} > \Pi^*_{\alpha\to 0} = 0$. Using the definition of $\bar{\tau}_\alpha$, rewrite Π^*_α as

$$(1-\alpha)(\min\{\bar{\tau}_\alpha, b\} + \bar{\Delta}_\alpha)G(v + b - \min\{\bar{\tau}_\alpha, b\}) + \alpha(\bar{\Delta}_\alpha - \sigma)G(v + b + \sigma) - K(\bar{\Delta}_\alpha).$$

Recall $\bar{\tau}_\alpha$ is increasing in α. If $\bar{\tau}_\alpha < b$, then the derivative is

$$\frac{d\Pi_\alpha^*}{d\alpha} = -(\bar{\tau}_\alpha + \bar{\Delta}_\alpha)G(v + b - \bar{\tau}_\alpha) + (\bar{\Delta}_\alpha - \sigma)G(v + b + \sigma) + (1-\alpha)\frac{d}{d\alpha}\left[(\bar{\tau}_\alpha + \bar{\Delta}_\alpha)G(v + b - \bar{\tau}_\alpha)\right],$$

where we used the envelope theorem on $\bar{\Delta}_\alpha$. Then

$$\frac{d\Pi_\alpha^*}{d\alpha}|_{\alpha=1} = -(\bar{\tau}_\alpha + \bar{\Delta}_\alpha)G(v + b - \bar{\tau}_\alpha) + (\bar{\Delta}_\alpha - \sigma)G(v + b + \sigma) < 0$$

due to $\bar{\tau}_\alpha > \Delta^l - \sigma > -\sigma$ and (2). Thus, $\Pi^*_{\alpha=1} < \Pi^*_{\alpha\in(0,1)}$. Finally, setting $\alpha \to 0$ gives $\bar{\tau}_\alpha = 0$, so $\Pi^*_{\alpha=0} = 0$.

Endogenous commitment. If M does not commit not to imitate, it is as if $\alpha \to 0$ and M's profit is $\max\{0, (b + \max\{\sigma, \Delta^l\})G(v)\}$. Thus, M weakly prefers imitating given $\Pi^*_{\alpha\in(0,1)} > \Pi^*_{\alpha=1} > \Pi^*_{\alpha\to 0}$.

E Imperfect and value-adding imitation

Consider the same analysis as in Section 4 but M's imitation is imperfect. For each given Δ chosen by S, the value of M's imitated product is $v + \Delta - \epsilon$, where $\epsilon > 0$ indicates imperfect imitation and $\epsilon < 0$ indicates value-adding imitation. To focus on the interesting case where imitation is relevant, we assume $\epsilon < \min\left\{\Delta^l, \Delta^l - \sigma\right\}$, so that the imitated product is better than both the fringe's product and M's original product without imitation. We further assume that G is weakly concave and that ϵ is not too large such that

$$\epsilon < \frac{K'(\Delta^l)}{g(v+b+\Delta^l)}.$$

As will be shown below, this assumption implies that S has zero incentive to innovate whenever it expects its product will be imitated.

Obviously, the no-imitation pricing subgame in dual mode remains unchanged. Consider the post-imitation pricing subgame. Denote

$$\tilde{\Pi}_{imi}^{exploit} = (\min\{\tau, b\} + \Delta - \epsilon)G(v + b - \min\{\tau, b\})$$

and let $\tilde{p}_i^{show}$ be the solution of

$$\tau G(v + \Delta + b - \tilde{p}_i^{show}) = \tilde{\Pi}_{imi}^{exploit}. \tag{E.1}$$

Then, the post-imitation pricing subgame has two equilibria:

- *Exploitative equilibrium (with ϵ imitation).* M sells to all consumers, with prices $p_m^* = \min\{\tau, b\} + \Delta - \epsilon$, $p_i^* = \tau$, and $p_o^* \geq p_i^* - b$. Profits are $\Pi = \tilde{\Pi}_{imi}^{exploit}$ and $\pi = -K(\Delta)$. The equilibrium exists if and only if $\tilde{p}_i^{show} \leq \tau$ or $\epsilon < 0$ or $\tau > b + \Delta$.

- *Price squeeze equilibrium (with ϵ imitation).* S sells to all consumers. Any price profile satisfying $p_i^* = \min\{\tilde{p}_i^{show}, p_m^* + \epsilon\}$, $p_o^* \geq p_i^* - b$, and $p_m^* \in [\max\{\tau - \epsilon, 0\}, \min\{\tau, \tau + \Delta - \epsilon, b + \Delta - \epsilon\}]$ is an equilibrium. Our equilibrium selection rule selects the lowest p_m^*, so

  $$p_i^* = \min\{\tilde{p}_i^{show}, \max\{\tau, \epsilon\}\}.$$

  Profits are

  $$\tilde{\Pi}_{imi}^{sqz} = \max\left\{\tau G(v + \Delta + b - \max\{\tau, \epsilon\}), \tilde{\Pi}_{imi}^{exploit}\right\}$$

  and

  $$\pi = \max\{\epsilon - \tau, 0\} G(v + b + \Delta - \epsilon) - K(\Delta). \tag{E.2}$$

  The equilibrium exists if and only if $\tilde{p}_i^{show} \geq \tau$ and $\epsilon \geq 0$ and $\tau \leq b + \Delta$.

Consider innovation and imitation decisions in stage 2. If $\epsilon < 0$, and given the assumption of $\epsilon < \Delta - \sigma$, it is easy to verify that M always strictly prefers imitating to inducing the exploitative equilibrium, regardless of τ. Thus, S always chooses Δ^l.

Suppose $\epsilon > 0$. We note that M always strictly prefers imitation, except when τ is such that in the no-imitation pricing subgame the commission induces the price squeeze equilibrium with $p_i^* = \tau$ (so that

M's profit is the same with and without imitation). Thus, if imitation does not arise, S's profit must be $\pi = -K(\Delta)$. If imitation arises, S's profit is at most (E.2). The assumption of ϵ being not too large implies that S always chooses Δ^l, regardless of whether imitation arises or not.

Given that S always chooses Δ^l, it follows that M optimally chooses $\tau = b + \Delta^l$.

Lemma E.1 *Suppose $\epsilon < \Delta^l - \sigma$. In equilibrium, M sets $\tau^{dual} = b + \Delta^l$, S participates and sets $\Delta^{dual} = \Delta^l$.*

- *If $\epsilon > 0$ (imperfect imitation), M does not imitate. The prices are $p_i^* = \tau$, $p_o^* = 0$, and $p_m^* = \tau$, and S sells to all consumers through the marketplace.*

- *If $\epsilon < 0$ (value-adding imitation), M imitates. The prices are $p_i^* = \tau$, $p_o^* = 0$, and $p_m^* = b + \Delta^l - \epsilon$, and M sells to all consumers.*

The profits are $\Pi^{dual} = (b + \Delta^l + \max\{-\epsilon, 0\})G(v)$ and $\pi^{dual} = 0$.

This equilibrium characterization has the same structure as Proposition 5, so that all the subsequent analysis in Section 4 continues to apply.

F Imperfect steering

Consumers differ in the information they have regarding the offerings available on M's marketplace. A fraction $\lambda > 0$ of consumers are "searchers" and they are aware of S's existence as long as S is available on M's marketplace (as in the baseline model). The remaining fraction $1 - \lambda$ of consumers are "non-searchers" and they rely on M's recommendation to discover products so that M can manipulate their awareness of S's existence. Specifically, after all prices are set, M makes a binary choice on whether to show S's product so that non-searchers also become aware of it.

For tractability, we adopt the timing in Section C. We assume that S's innovation cost function satisfies

$$K'(\Delta^l) \geq \lambda. \tag{F.1}$$

As will be seen below, this assumption implies that S always chooses the lowest possible innovation level whenever it anticipates product imitation or a price squeeze by M in the pricing subgame of the dual mode. Relaxing this assumption shifts the welfare and consumer surplus comparisons in favor of the dual mode.

We focus on $\sigma = 0$ to simplify the discussion (the analysis is easily extendable to the case of $\sigma \neq 0$). Recall that $\sigma \neq 0$ primarily affects M's choice of mode after the dual mode is banned, and that M is indifferent between pure marketplace mode and pure seller mode when $\sigma = 0$.

No-imitation pricing subgame. For the pricing subgame, we first consider the case where M has chosen not to imitate S's product.

Lemma F.1 *(No-imitation subgame).*

- *If $\tau \leq b$, then M sets $p_m = \max\{\tau - \Delta, 0\}$, S sets $p_i = p_m + \Delta$, $p_o > p_i - b$ and sell to all consumers through M, and profits are $\Pi = \tau G(v + b - p_m)$ and $\pi = \max\{(\Delta - \tau)G(v + b + \Delta - \tau), 0\} - K(\Delta)$.*

- *If $\tau \in (b, b + \Delta(1 - \lambda)]$, then M sets*

$$p_m = \max\{b - \Delta + \frac{\tau - b}{1 - \lambda}, 0\},$$

S sets $p_i = p_m + \Delta$, $p_o > p_i - b$ and sells to all consumers through M, and profits are $\Pi = \tau G(v + b - p_m)$ and $\pi = \max\left\{\lambda(\frac{\tau - b}{1-\lambda})G(v + \Delta - \frac{\tau - b}{1-\lambda}), (\Delta - \tau)G(v + b - \tau)\right\}$.

- *If $\tau > b + \Delta(1 - \lambda)$, one of the following is the equilibrium: (i) M sets $p_m = b - \Delta$ to sell to all consumers, S sets $p_o = \Delta$, $p_i > p_o + b$, and profits are $\Pi = (b - \Delta)G(v + \Delta)$ and $\pi = 0$; (ii) M sets $p_m = b$ to sell to all non-searchers, S sets $p_o = \Delta$, $p_i > p_o + b$ to sell to all searchers directly, and profits are $\Pi = b(1 - \lambda)G(v)$ and $\pi = \Delta\lambda G(v)$.*

Proof. Consider $\tau \leq b$. We know that S never prefers selling directly, i.e., $p_o > p_i - b$. For all $p_m < \tau - \Delta$, S cannot profitably make any sales inside or outside, so it sets $p_i = \tau$. For all $p_m \in [\tau - \Delta, \tau]$, S sell to all consumers inside at $p_i = p_m + \Delta$ and it will be shown by M to non-searchers (given that $p_m < \tau$ and that the purchase probability is the same at $G(v + b - p_m)$). For all $p_m > \tau$, consumers never purchase M's product and the analysis is the same as for the pure marketplace mode, in which S optimally sets $p_i = \tau + \Delta$ to sell to all consumers through the marketplace. Anticipating S's pricing responses, M's profit as a function of p_m is

$$\Pi^{no-imi}_{\tau \leq b}(p_m) = \left\{ \begin{array}{ll} p_m G(v + b - p_m) & \text{if } p_m < \tau - \Delta \\ \tau G(v + b - p_m) & \text{if } p_m \in [\tau - \Delta, \tau] \\ \tau G(v + b - \tau) & \text{if } p_m > \tau \end{array} \right\}.$$

Assumption (2) implies that $p_m = \tau - \Delta$ is optimal, subject to the constraint $p_m \geq 0$. The resulting profit is $\tau G(v + b - \max\{\tau - \Delta, 0\})$.

Consider $\tau \in (b, b + \Delta(1 - \lambda)]$. For all $p_m \leq b - \Delta$, S cannot profitably make any sales inside or outside the marketplace. For all $p_m \in (b - \Delta, \tau - \Delta]$, S sets $p_o = p_m + \Delta - b$ and $p_i > p_o + b$ to all searchers directly given that it will incur losses for any inside sales. For $p_m \in (\tau - \Delta, b]$, S has two pricing options. First, it can sell to all searchers directly, earning

$$\pi^{out}(p_m + \Delta - b) = (p_m + \Delta - b)G(v + b - p_m)\lambda.$$

Second, it can set $p_i = p_m + \Delta$ and sell to all consumers, earning

$$\pi^{in}(p_m + \Delta) = (p_m + \Delta - \tau)G(v + b - p_m).$$

Equating S's profit for both options and solving the indifference condition, S prefers selling directly if and only if $p_m < b - \Delta + \frac{\tau - b}{1-\lambda}$, and it prefers selling through M if and only if $p_m \geq b - \Delta + \frac{\tau-b}{1-\lambda}$. Moreover, note that $b - \Delta + \frac{\tau-b}{1-\lambda} \leq b$ given $\tau \leq b + \Delta(1-\lambda)$. For all $p_m > b$, consumers never purchase M's product and the analysis is the same as in the pure marketplace mode. Anticipating S's pricing responses, M's profit as a function of p_m is

$$\Pi^{no-imi}_{\tau \in (b, b+\Delta(1-\lambda)]}(p_m) = \left\{ \begin{array}{cl} p_m G(v + b - p_m) & \text{if } p_m \leq b - \Delta \\ p_m(1-\lambda)G(v + b - p_m) & \text{if } p_m \in (b - \Delta, b - \Delta + \frac{\tau - b}{1-\lambda}) \\ \tau G(v + b - p_m) & \text{iif } p_m \in [b - \Delta + \frac{\tau-b}{1-\lambda}, b] \\ 0 & \text{if } p_m > b \end{array} \right\}.$$

Assumption (2) implies that both the first and the second rows are maximized at their respective upperbounds, attaining values $(b-\Delta)G(v+\Delta)$ and $(\tau + (b - \Delta)(1-\lambda) - b)\, G\left(v + \Delta + b - \frac{\tau - b\lambda}{1-\lambda}\right)$, both of which are smaller than the maximum of the third row:

$$\tau G\left(v + b - \max\{b - \Delta + \frac{\tau - b}{1-\lambda}, 0\}\right).$$

Thus, M chooses $p_m = \max\{b - \Delta + \frac{\tau-b}{1-\lambda}, 0\}$.

Consider $\tau > b + \Delta(1-\lambda)$. Following the analysis above,

$$\Pi^{no-imi}_{\tau > b+\Delta(1-\lambda)}(p_m) = \left\{ \begin{array}{cl} p_m G(v + b - p_m) & \text{if } p_m \leq b - \Delta \\ p_m(1-\lambda)G(v + b - p_m) & \text{if } p_m \in (b - \Delta, b) \\ 0 & \text{if } p_m > b \end{array} \right\},$$

so M sets $p_m = b - \Delta$ or b. ■

Post-imitation pricing subgame. After imitation, M's offering is now valued at $b + \Delta$.

Lemma F.2 *(Post-imitation subgame).*

- *If $\tau \leq b$, then M sets $\hat{p}_m$ implicitly defined by*

$$\hat{p}_m = \arg \max_{p_m \in (\tau, \tau+\Delta]} \{(\lambda\tau + (1-\lambda)p_m)G(v + b + \Delta - p_m)\}$$

and sells to all non-searchers, S sets $p_i = \hat{p}_m$, $p_o > p_i - b$ and sells to all searchers through M, *and profits are*

$$\hat{\Pi} = \max_{p_m \in (\tau, \tau+\Delta]} \{(\lambda\tau + (1-\lambda)p_m)G(v+b+\Delta-p_m)\} \tag{F.2}$$

and $\pi = (\hat{p}_m - \tau)\lambda G(v+b+\Delta-\hat{p}_m)$.

- *If* $\tau > b$, *one of the following is the equilibrium: (i)* M *sets* $p_m = b$ *to sell to all consumers,* S *sets* $p_o = 0$ *and* $p_i > p_o + b$, *and profits are* $\Pi = bG(v+\Delta)$ *and* $\pi = 0$; *(ii)* M *sets* $p_m = b+\Delta$ *to sell to all non-searchers,* S *sets* $p_o = \Delta$ *and* $p_i > p_o + b$ *to sell to all searchers directly, and profits are* $\Pi = (b+\Delta)(1-\lambda)G(v)$ *and* $\pi = \Delta\lambda G(v)$.

Proof. Consider $\tau \leq b$. We know that S never prefers selling directly, i.e., $p_o > p_i - b$. For all $p_m \leq \tau$, S cannot profitably make any sales inside or outside so it sets $p_i = \tau$. For all $p_m \in (\tau, \tau+\Delta]$, S will not get recommended for all $p_i \geq \tau$ because

$$\tau G(v+\Delta+b-\tau) < p_m G(v+\Delta+b-p_m)$$

given (12). Thus, S slightly undercuts M and sells to all searchers through the marketplace. For all $p_m > \tau+\Delta$, consumers never purchase M's product and the analysis is the same as the pure marketplace mode, in which S optimally sets $p_i = \tau+\Delta$ to sell to all consumers inside. Anticipating S's pricing responses, M's profit as a function of p_m is

$$\Pi^{imi}_{\tau \leq b}(p_m) = \begin{cases} p_m G(v+b+\Delta-p_m) & \text{if } p_m \leq \tau \\ (\lambda\tau + (1-\lambda)p_m)G(v+b+\Delta-p_m) & \text{if } p_m \in (\tau, \tau+\Delta] \\ \tau G(v+b-\tau) & \text{if } p_m > \tau+\Delta \end{cases}.$$

M optimally sets $p_m = \tau+\Delta$ because the first row is increasing in p_m given (12). Hence, continuity implies that M optimally sets $p_m \in [\tau, \tau+\Delta]$ and solves

$$\max_{p_m \in (\tau, \tau+\Delta]} \{(\lambda\tau + (1-\lambda)p_m)G(v+b+\Delta-p_m)\}.$$

Consider $\tau > b$. First note that for all $p_m \leq b+\Delta$, we know that S never sells to non-searchers. If $p_m < \tau$, then getting recommended and purchased requires S to set $p_i < \tau$. If $p_m = \tau$, selling to non-searchers requires S to set $p_i = \tau$, which results in zero profit, but S would then strictly prefer inducing showrooming and selling to non-searchers, earning margin $\tau - b > 0$. If $p_m > \tau$, note that for all $p_i \geq \tau$ we have

$$\begin{aligned} & \tau G(v+b+\Delta-p_i) \leq \tau G(v+b+\Delta-\tau) \\ < \quad & p_m G(v+b+\Delta-p_m), \end{aligned}$$

so M never shows S. Hence, there is no downside if S induces searchers to purchase directly, and so in any equilibrium S must be selling through the direct channel. For all $p_m > b + \Delta$, consumers never purchase M's product and the analysis is the same as in the pure marketplace mode whereby S optimally chooses to sell to all consumers directly (given $\tau > b$, M's recommendation rule implies S will be shown to both searchers and non-searchers). So $p_i > p_o + b$ in equilibrium. Then, following a similar analysis as in the previous case (and taking into account that S always sells outside), we can derive M's profit as a function of p_m:

$$\Pi^{imi}_{\tau>b}(p_m) = \begin{cases} p_m G(v+b+\Delta-p_m) & \text{if } p_m \leq b \\ p_m(1-\lambda)G(v+b+\Delta-p_m) & \text{if } p_m \in (b, b+\Delta] \\ 0 & \text{if } p_m > b+\Delta \end{cases} .$$

Assumption (12) implies that both the first and second rows of $\Pi^{imitate}_{\tau>b}$ are maximized at the upper-bounds $p_m = b$ and $p_m = b + \Delta$, and M will choose between these two prices. ■

Imitation decision. For each given Δ set by S, we do a side-by-side comparison on M's profit with and without imitation

	Π^{no-imi}	Π^{imi}
$\tau \leq b$	$\tau G(v+b-\max\{\tau-\Delta, 0\})$	(F.2)
$\tau \in (b, b+\Delta(1-\lambda)]$	$\tau G(v+b-\max\{b-\Delta+\frac{\tau-b}{1-\lambda}, 0\})$	$\max\{bG(v+\Delta), (b+\Delta)(1-\lambda)G(v)\}$
$\tau > b+\Delta(1-\lambda)$	$\max\{bG(v), b(1-\lambda)G(v)\}$	$\max\{bG(v+\Delta), (b+\Delta)(1-\lambda)G(v)\}$

If $\tau > b + \Delta(1-\lambda)$, then $\Pi^{imi} > \Pi^{no-imi}$ obviously. If $\tau \leq b$, then $\Pi^{imi} > \Pi^{no-imi}$ given the definition of (F.2). For $\tau \in (b, b + \Delta(1-\lambda)]$, imitation does not occur if and only if $\Pi^{no-imi} \geq \Pi^{imi}$, or (F.5), which is defined below, holds.

Innovation decision. Recall that $\bar{\Delta}$ is defined by $G(v+b) = K'(\bar{\Delta})$, and it is the highest possible innovation level that would arise in any equilibrium. Define

$$\pi_1 = \max_{\Delta} \{(p_m^* + \Delta - \tau)G(v+b+\Delta-p_m^*) - K(\Delta)\} \tag{F.3}$$

subject to

$$p_m^* = \max\{b - \Delta + \frac{\tau-b}{1-\lambda}, 0\}$$

$$\tau \leq b + \Delta(1-\lambda) \tag{F.4}$$

$$\tau G(v+b-p_m^*) \geq \max\{bG(v+\Delta), (b+\Delta)(1-\lambda)G(v)\}, \tag{F.5}$$

which represents S's maximized profit subject to the constraint that Δ is such that M does not imitate

S's product. If the parameters are such that the set of Δ satisfying the constraints is empty, we set $\pi_1 = -\infty$ without loss of generality. Define

$$\pi_2 = \begin{cases} 0 & \text{if } bG(v+\Delta^l) \geq (b+\Delta^l)(1-\lambda)G(v) \\ \Delta^l \lambda G(v) & \text{if } bG(v+\Delta^l) \leq (b+\Delta^l)(1-\lambda)G(v) \end{cases},$$

which represents S's maximized profit if its product is imitated by M. Recall (F.1) implies that S always chooses Δ^l whenever it anticipates that its product will be imitated.

Lemma F.3 *Consider S's innovation decision. If $\bar{\Delta} \leq b$, then S chooses Δ^l regardless of τ. If $\bar{\Delta} > b$, then*

- *If $\tau \notin (b, b\lambda + \bar{\Delta}(1-\lambda))$, then S chooses Δ^l and M imitates.*
- *If $\tau \in (b, b\lambda + \bar{\Delta}(1-\lambda))$ and $\pi_1 < \pi_2$, then S chooses Δ^l and M imitates.*
- *If $\tau \in (b, b\lambda + \bar{\Delta}(1-\lambda))$ and $\pi_1 \geq \pi_2$, then S chooses the constrained maximizer of (F.3) and M does not imitate. If, in addition, λ is sufficiently small, then S always chooses Δ^l.*

Proof. For all $\tau \leq b$, we know that M always imitates, so S chooses Δ^l. If $\tau \geq b\lambda + \bar{\Delta}(1-\lambda)$, we have $p_m^* \geq 0$ for all Δ, so S's profit is either $\pi_1 = \max_\Delta \lambda(\frac{\tau-b}{1-\lambda})G(v+\Delta-\frac{\tau-b}{1-\lambda}) - K(\Delta)$ (if not imitated) or π_2 (if imitated). Assumption (F.1) implies that S always chooses Δ^l in both cases.

It remains to consider $\tau \in (b, b\lambda + \bar{\Delta}(1-\lambda))$. If $\bar{\Delta} \leq b$, then this set is empty and so it suffices to focus on $\bar{\Delta} > b$. In this case, S solves the maximization problems π_1 and π_2, as stated in the lemma. To prove the last part, suppose

$$\lambda \leq 1 - \frac{bG(v+\Delta^l)}{(b+\Delta^l)G(v)} \tag{F.6}$$

so that $\max\{bG(v+\Delta), (b+\Delta)(1-\lambda)G(v)\} = (b+\Delta)(1-\lambda)G(v)$ for all Δ. Let us focus on the maximization problem π_1. For all $\frac{\tau-b}{1-\lambda} \leq \Delta < \frac{\tau-b\lambda}{1-\lambda}$, we have $p_m^* = b - \Delta + \frac{\tau-b}{1-\lambda}$ so that objective (F.3) becomes

$$\lambda(\frac{\tau-b}{1-\lambda})G(v+\Delta-\frac{\tau-b}{1-\lambda}) - K(\Delta),$$

with derivative

$$\begin{aligned} & \lambda\left(\frac{\tau-b}{1-\lambda}\right)g(v+\Delta-\frac{\tau-b}{1-\lambda}) - K'(\Delta) \\ = \; & \lambda\left(\frac{\tau-b}{1-\lambda}\right)\frac{g(v+\Delta-\frac{\tau-b}{1-\lambda})}{G(v+\Delta-\frac{\tau-b}{1-\lambda})}G(v+\Delta-\frac{\tau-b}{1-\lambda}) - K'(\Delta) \\ \leq \; & \lambda\Delta\frac{g(v)}{G(v)}G(v+\Delta-\frac{\tau-b}{1-\lambda}) - K'(\Delta) \\ < \; & \lambda - K'(\Delta) \\ < \; & 0, \end{aligned}$$

where the first inequality is due to $\frac{\tau-b}{1-\lambda} \leq \Delta$ and log-concavity of G; the second inequality is due to $G(v+\Delta-\frac{\tau-b}{1-\lambda}) < 1$ and (12); and the final inequality is due to (F.1). If $\Delta^l > \frac{\tau-b}{1-\lambda}$, then all $\Delta \in \left[\Delta^l, \frac{\tau-b\lambda}{1-\lambda}\right)$ are dominated by Δ^l and $\Delta < \Delta^l$ are infeasible. If $\Delta^l \leq \frac{\tau-b}{1-\lambda}$, then all $\Delta \in \left[\frac{\tau-b}{1-\lambda}, \frac{\tau-b\lambda}{1-\lambda}\right)$ are dominated by $\frac{\tau-b}{1-\lambda}$, whereby $\pi = \lambda(\frac{\tau-b}{1-\lambda})G(v) - K(\frac{\tau-b}{1-\lambda})$. A change of variable argument and (12) imply that this profit is lower than $\Delta^l\lambda G(v)$, which is what S earns from setting Δ^l and inducing imitation due to (F.6). If $\Delta \geq \frac{\tau-b\lambda}{1-\lambda}$, then imitation occurs if and only if

$$\tau G(v+b) < (b+\Delta)(1-\lambda)G(v),$$

which is implied by

$$\lambda < 1 - \frac{bG(v+b)}{(b+\bar{\Delta})G(v) + (b-\bar{\Delta}))G(v+b)}. \tag{F.7}$$

In such cases, S's profit is $\Delta\lambda G(v) - K(\Delta)$, which is decreasing in Δ, so S chooses Δ^l. We conclude that if λ is small such that (F.6) and (F.7) are both satisfied, then S chooses Δ^l. ■

Commission decision. We focus on the two cases where S's innovation decision has a closed-form solution equal to Δ^l: (i) $\bar{\Delta} \leq b$; (ii) $\bar{\Delta} > b$ and λ is sufficiently small.

Lemma F.4 *(Dual mode equilibrium with self-preferencing and imitation). If $\bar{\Delta} \leq b$, then both configurations below are equilibria:*

- *M sets $\tau^{dual} = b$, S participates, setting Δ^l, and M imitates. The prices are $p_i^* = p_m^*$, $p_o^* = 0$, and $p_m^* = \hat{p}_m$ where*

$$\hat{p}_m = \arg \max_{p_m \in (b, b+\Delta^l]} \left\{(\lambda b + (1-\lambda)p_m)G(v+b+\Delta^l - p_m)\right\}, \tag{F.8}$$

  M sells to all non-searchers whereas S sells to all searchers.

- *M sets $\tau^{dual} = \lambda b + (1-\lambda)\hat{p}_m$, S participates, setting Δ^l, and M does not imitate. The prices are $p_i^* = p_m^* + \Delta^l$, $p_o^* = 0$, and $p_m^* = b - \Delta^l + \frac{\tau-b}{1-\lambda}$, and S sells to all consumers.*

Equilibrium profits are $\Pi^{dual} = (\lambda b + (1-\lambda)\hat{p}_m)G(v+b+\Delta^l - \hat{p}_m)$ and $\pi^{dual} = \lambda(\hat{p}_m - b)G(v)$. If instead $\bar{\Delta} > b$ and λ is sufficiently small, then both configurations below are equilibria:

- *M sets $\tau^{dual} = b$, S participates and sets Δ^l, and M imitates. The prices are $p_i^* = p_m^*$, $p_o^* = 0$, and $p_m^* = b + \Delta^l$ where M sells to all non-searchers whereas S sells to all searchers.*

- *M sets $\tau^{dual} = b + (1-\lambda)\Delta^l$, S participates and sets Δ^l, and M does not imitate. The prices are $p_i^* = p_m^* + \Delta^l$, $p_o^* = 0$, and $p_m^* = b - \Delta^l + \frac{\tau-b}{1-\lambda}$, and S sells to all consumers.*

Equilibrium profits are $\Pi^{dual} = (b + (1-\lambda)\Delta^l)G(v)$ and $\pi^{dual} = \lambda\Delta^l G(v)$.

Proof. Suppose $\bar{\Delta} \leq b$. For $\tau \leq b$, M necessarily imitates so

$$\Pi = \max_{p_m \in (\tau, \tau+\Delta^l]} \left\{ (\lambda\tau + (1-\lambda)p_m) G(v + b + \Delta^l - p_m) \right\}, \tag{F.9}$$

and $\tau = b$ is optimal in this range by the envelope theorem. For $\tau \in (b, b + \Delta^l(1-\lambda)]$, M's maximized profit is

$$\Pi = \max_{\tau \in (b, b+\Delta^l(1-\lambda)]} \tau G(v + \Delta^l - \frac{\tau - b}{1-\lambda}), \tag{F.10}$$

where we have used $\max\{b - \Delta + \frac{\tau-b}{1-\lambda}, 0\} = b - \Delta + \frac{\tau-b}{1-\lambda}$ because $\tau \geq b \geq \bar{\Delta}$. A change of variable argument implies that (F.10) is equivalent to (F.9). Hence, (F.10) is higher than the post-imitation profits

$$\Pi^{imi}_{\tau>b} = \max\{bG(v + \Delta), (b + \Delta)(1-\lambda)G(v)\},$$

so M indeed does not imitate in equilibrium. Likewise, all $\tau > b + \Delta^l(1-\lambda)$ leads to $\Pi^{imi}_{\tau>b}$ lower than (F.9) and (F.10).

Suppose $\bar{\Delta} > b$ and λ is sufficiently small. For $\tau \leq b$, M necessarily imitates so its profit is (F.9) and $\tau = b$ is optimal in this range. Moreover, if λ is sufficiently small such that

$$\lambda \leq \frac{G(v)/g(v) - \Delta^l - b}{G(v)/g(v) - \Delta^l}, \tag{F.11}$$

then (F.9) is maximized at $\tau = b$ and $p_m = b + \Delta^l$ because

$$(\lambda b + (1-\lambda)p_m)G(v + b + \Delta^l - p_m)$$

would be increasing in p_m for all $p_m \leq \tau + \Delta^l$. If $\tau \in (b, b + \Delta^l(1-\lambda)]$, then M's maximized profit is (F.10), and (F.11) implies $\tau = b + \Delta^l(1-\lambda)$ is optimal. ■

Compared to the baseline setup with perfect steering (Proposition 5), Lemma F.4 shows that imperfect steering leads to a few key differences.

We first focus on the equilibrium with $\tau = b$ and M imitating. In this case, imperfect steering enables S to sell to a positive fraction of consumers whenever M chooses exploitative pricing. As a result, there is a transfer of surplus from M to S, so S's profit is strictly higher than in the case with perfect steering. Nonetheless, provided that λ is not too large, (F.8) implies $p_m^* = b + \Delta^l$ so that the pricing behaviors of M and S are unaffected. The final consumer price remains as high as in the case with perfect steering. Once λ becomes sufficiently large, the mass of non-searchers is small such that the exploitative pricing is no longer optimal. Specifically, this requires

$$\lambda > \frac{G(v)/g(v) - \Delta^l - b}{G(v)/g(v) - \Delta^l}.$$

Lowering the price induces S to set a lower price, which increases the transaction volume and hence the commission revenue, and so $p_m^* \in [b, b + \Delta^l)$. When $\lambda \to 1$, we have $p_m^* \to b$, which is similar to the case where self-preferencing is banned (i.e., the description above Proposition 7).

Consider the equilibrium with $\tau > b$ and M not imitating. In this case, imperfect steering partially restores the showrooming constraint (relative to the case of perfect steering) so that M's optimal fee is strictly below $b+\Delta^l$. Consequently, there is again a surplus transfer from M to S. Imperfect steering also affects the mechanism of a price squeeze, whereby the extent of the price squeeze that M can implement is now limited by the possibility of S inducing showrooming to sell to non-searchers. This means p_m is such that S is indifferent between inducing and not inducing showrooming, i.e.,

$$(p_m + \Delta - b)G(v + b - p_m)\lambda = (p_m + \Delta - \tau)G(v + b - p_m)$$

or

$$p_m = b - \Delta + \frac{\tau - b}{1 - \lambda}.$$

This leads to an interesting feature whereby an increase in τ undermines the extent of the price squeeze. Specifically, whenever M raises τ by one unit, it has to increase p_m by $\frac{1}{1-\lambda}$ units to prevent S from inducing showrooming. Consequently, the showrooming constraint on τ does not necessary bind in equilibrium.

Banning dual mode. We know that if (i) $\bar{\Delta} \leq b$ and $\lambda \leq \frac{G(v)/g(v)-\Delta^l-b}{G(v)/g(v)-\Delta^l}$, or (ii) $\bar{\Delta} > b$ and λ is sufficiently small, then the equilibrium characterization in Lemma F.4 has the final consumer price at $b + \Delta^l$ so that it is broadly similar to Proposition 5. Banning the dual mode leads to Proposition 6, except that π would strictly decrease whenever M switches to the seller mode after the ban.

If $\bar{\Delta} \leq b$ and $\lambda > \frac{G(v)/g(v)-\Delta^l-b}{G(v)/g(v)-\Delta^l}$, then the equilibrium in Lemma F.4 involves a final price strictly below $b + \Delta^l$. In this case, we have

M's choice of mode	Π	π	CS	Δ	W
Seller	$\downarrow$	$\downarrow$	$\downarrow$	.	$\downarrow$
Marketplace	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	ambiguous

"." = not changing; " $\uparrow$ " = increasing; " $\downarrow$ " = decreasing.

Hence, imperfect steering in dual mode implies that the ban on the dual mode is more likely to harm consumers. Meanwhile, the ambiguous welfare change in the case of switching to the marketplace mode reflects the trade-off between fewer transactions and the higher innovation level.

G Comparison with the wholesaler-retailer model

In this section, we analyze the wholesaler-retailer model described in Section 5 of the main text. We consider two possibilities: (i) wholesale prices are set by third-party suppliers; (ii) wholesale prices are set by M.

G.1 Third-party suppliers set wholesale prices

Formally, the timing is the following:

1. M chooses its mode of operation;
2. S chooses its innovation level and wholesale price, and all fringe sellers set their wholesale prices simultaneously;
3. All sellers, including S and M, set retail prices simultaneously;
4. After observing the existence of S if it is sold by M, all consumers make their purchase decisions.

In what follows, we denote M's price for S's product as p_m^s, M's price for the fringe product as p_m^f, M's price for its in-house product as p_m^h, and S's price at its direct channel as p_o. Similar to the main text, whenever there are multiple equilibria in any subgame that are payoff-ranked by M, we select the one preferred by M. Then, whenever there are multiple equilibria in any subgame that are payoff-equivalent for M, but payoff-ranked by S, we select the one preferred by S.

Third-party product mode. Whenever M does not sell S's product, the only alternative available is the fringe suppliers' product, which is priced at a wholesale price of zero. In this case, M's retail price is only constrained by the competition with fringe suppliers. If M obtains S's product, consumers are aware of S's direct channel so that M's retail price is constrained by the competition with S's direct channel (whereby S has an opportunity cost w whenever it tries to undercut M). For any given wholesale price w set by S in stage 2, it is easy to solve for the following equilibrium in stage 3 using (2).

- If $w > \Delta$, then M does not obtain S's product. The equilibrium prices are $p_m^f = b$ and p_o taking any value. M sells to all consumers. Profits are $\Pi = bG(v)$ and $\pi = -K(\Delta)$.
- If $w \leq \Delta$, then M obtains S's product. The equilibrium prices are $p_m^s = b + w$ and $p_o = w$. M sells to all consumers. Profits are $\Pi = bG(v + \Delta - w)$ and $\pi = wG(v + \Delta - w) - K(\Delta)$.

In stage 2, S solves $\max_{w \leq \Delta} wG(v + \Delta - w)$, and (2) implies that $w = \Delta$ is optimal. Then, S chooses innovation level Δ^{3rd} that solves the first-order condition

$$G(v) = K'(\Delta^{3rd}).$$

In the overall equilibrium, profits are $\Pi = bG(v)$ and $\pi = \Delta^{3rd}G(v) - K(\Delta^{3rd})$.

In-house product mode. In this mode, M is selling its in-house product only so consumers are unaware of S's product. Given (2), M's equilibrium price is bound by the competition with the fringe suppliers' products in their direct channel. The overall equilibrium is:

- S chooses innovation level $\Delta^{in\text{-}house} = \Delta^l$. The equilibrium prices are $p_m^h = b + \sigma > 0$, whereas p_o and w take any values. M sells to all consumers. Profits are $\Pi = (b + \sigma)G(v)$ and $\pi = 0$.

Dual-product mode. Relative to the third-party product mode, the possibility of M selling its own product additionally bounds the wholesale price that it is willing to accept to obtain S's product. For each given Δ, define $\underline{w} = \underline{w}(\Delta)$ implicitly as the highest wholesale price where M is indifferent between selling S's product and selling its in-house product:

$$bG(v + \Delta - \underline{w}) = (b + \max\{\sigma, 0\})G(v). \tag{G.1}$$

Note that $\sigma \leq 0$ implies $\underline{w} = \Delta$ and $\sigma > 0$ implies $\underline{w} < \Delta - \sigma$ (due to (2)). Note that $d\underline{w}/d\Delta = 1$. Then, similar to the analysis of the third-party product mode, for any given wholesale price w set by S in stage 2 we can solve for the following equilibrium:

- If $w > \underline{w}$, then M does not obtain S's product. If $\sigma < 0$, then M sells the fringe product, with equilibrium prices $p_m^f = b$ and p_o taking any value, and M selling to all consumers. If $\sigma \geq 0$, then M sells the in-house product, with equilibrium prices $p_m^h = b + \sigma$ and p_o taking any value, and M selling to all consumers. Profits are $\Pi = (b + \max\{\sigma, 0\})G(v)$ and $\pi = -K(\Delta)$.
- If $w \leq \underline{w}$, then M obtains S's product. The equilibrium prices are $p_m^s = b + w$ and $p_o = w$, and M sells to all consumers. Profits are $\Pi = bG(v + \Delta - w)$ and $\pi = wG(v + \Delta - w) - K(\Delta)$.

In stage 2, S solves $\max_{w \leq \underline{w}} wG(v + \Delta - w)$, and (2) implies that $w = \underline{w}$ is optimal as long as $\underline{w} \geq 0$ (otherwise $w = 0$ is optimal). Then, if $\sigma \leq 0$, then $\underline{w} = \Delta$ and S chooses innovation level Δ^{3rd}; if $\sigma > 0$, then S chooses Δ to maximize

$$\tilde{\pi}(\Delta) = \max\{\underline{w}(\Delta), 0\} G(v + \Delta - \underline{w}(\Delta)) - K(\Delta).$$

Let $\tilde{\Delta}$ be the solution of the first-order condition

$$G(v + \tilde{\Delta} - \underline{w}(\tilde{\Delta})) - K(\tilde{\Delta}),$$

where $\tilde{\Delta} > \Delta^{3rd}$. Then, we can summarize the overall equilibrium as:

- If $\sigma \leq 0$, then S sets Δ^{3rd} and wholesale price $w = \Delta^{3rd}$ and M sells S's product at $p_m^s = b + w$. The equilibrium profits are $\Pi = bG(v)$ and $\pi = \Delta^{3rd}G(v) - K(\Delta^{3rd})$.
- If $\sigma > 0$ and $\tilde{\pi}(\tilde{\Delta}) > 0$, then S sets $\tilde{\Delta}$ and wholesale price $w = \underline{w}(\tilde{\Delta})$ and M sells S's product at $p_m^s = b + w$. The equilibrium profits are $\Pi = bG(v)$ and $\pi = \underline{w}(\tilde{\Delta})G(v + \tilde{\Delta} - \underline{w}(\tilde{\Delta})) - K(\tilde{\Delta})$.
- If $\sigma > 0$ and $\tilde{\pi}(\tilde{\Delta}) \leq 0$, then S sets Δ^l and wholesale price $w = 0$ and M sells its in-house product at $p_m^h = b + \sigma$. The equilibrium profits are $\Pi = (b + \sigma)G(v)$ and $\pi = 0$.

Notice that M always weakly prefers the dual-product mode over the other two modes. For $\sigma > 0$, the condition of $\tilde{\pi}(\tilde{\Delta}) > 0$ can be alternatively stated as

$$\max_{\Delta} \underline{w}(\Delta)G(v + \Delta - \underline{w}(\Delta)) - K(\Delta) > 0$$
$$\text{subject to } (G.1).$$

By the envelope theorem, the maximized profit is decreasing in σ, hence the intermediate value theorem implies the existence of a threshold $\tilde{\sigma} > 0$ such that $\tilde{\pi}(\tilde{\Delta}) > 0$ if and only if $\sigma < \tilde{\sigma}$. Hence, banning the dual mode leads to the following result:

	M's choice of mode	Π	π	CS	Δ	W
if $\sigma \geq \tilde{\sigma}$	In-house	.	.	.	.	.
if $\sigma \in (0, \tilde{\sigma})$	In-house	.	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
if $\sigma \leq 0$	Third-party	.	.	.	.	.

"." = not changing; "$\uparrow$" = increasing; "$\downarrow$" = decreasing.

as stated in Proposition 10.

G.2 M sets wholesale prices

Suppose in stage 2, wholesale prices are determined by M after S chooses its innovation. Then, all third-party suppliers decide whether to supply to M at the given wholesale price. The analysis of the pricing subgames is the same as in the previous section. Clearly, the in-house product mode is unaffected by this modification because M does not obtain any products from third parties. Meanwhile, in the third-party product mode and the dual-product mode, M always chooses the lowest w subject to S's participation constraint, i.e., $w = 0$ (note that S's fixed innovation cost is sunk at this point and that consumers are unaware of S if it does not supply M). This implies that S has zero innovation incentive in these two modes, and we can summarize the equilibria as follows:

- **Third-party product mode.** In the overall equilibrium, S sets Δ^l, M sets $w = 0$ and sells S's product, and the prices are $p_m^s = b$ and $p_o = 0$. The profits are $\Pi = bG(v + \Delta^l)$ and $\pi = 0$.
- **Dual-product mode.** In the overall equilibrium, S sets Δ^l, M sets $w = 0$. If $\underline{w}(\Delta^l) > 0$, M sells S's product and the prices are $p_m^s = b$ and $p_o = 0$. The profits are $\Pi = bG(v + \Delta^l)$ and $\pi = 0$. If $\underline{w}(\Delta^l) \leq 0$, then M sells in-house product and the prices are $p_m^h = b + \sigma$ and $p_o = 0$. The profits are $\Pi = (b + \sigma)G(v)$ and $\pi = 0$.