Should platforms be allowed to sell on their own marketplaces?*

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Abstract

A growing number of platforms such as those run by Amazon, Apple and Google operate in a dual mode: running marketplaces, at the same time as selling products on them. We build a model to explore the implications of this controversial practice. We show that while banning the dual mode benefits third-party sellers, it often results in lower consumer surplus or total welfare, even after allowing for innovation by third-party sellers, and imitation and self-preferencing by the platform. Instead, policies that prevent platform imitation and self-preferencing always lead to better outcomes than an outright ban on the dual mode.

1 Introduction

An increasing number of e-commerce players such as Amazon, JD.com, Target, and Walmart, are acting both as marketplaces, i.e. enabling third party sellers to sell to consumers, and as sellers, i.e. selling products under their own name. Other notable examples include Apple's Appstore, Google's Playstore, Window's Apps, Intuit's Quickbooks Apps, Salesforce's AppExchange, and videogame consoles like Nintendo Switch, all of which sell their own apps alongside third-party apps. This practice has raised regulatory concerns over the lack of a level playing field, and led to investigations in Europe and the United States, with calls from various commentators and politicians for Amazon to be forced to separate its retail business from its marketplace. And in February 2019, India introduced new laws to force the separation of the two types of businesses, leading Amazon and the Walmart-backed Flipkart to change their business practices there.

In this paper we build a tractable model of a platform that can adopt a dual mode, in which it sells products in its own name (i.e. seller mode) alongside third-party sellers who sell competing products (i.e. marketplace mode) to explore the welfare implications of this practice. Specifically, we use the model to study how the platform's optimal choice of mode changes when

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the dual mode is outlawed, and derive the implications for consumer surplus and total welfare. We also conduct a similar analysis for several alternative policy options.

There are many reasons why it might be efficient (and pro-competitive) for the same platform to act as a seller for some types of products and as a marketplace for others. Most obviously, some types of products may be more efficiently provided by the platform, while others may be more efficiently provided by third-party sellers, and offering all of them in the same place provides one-stop shopping benefits to consumers. This means a blanket ban on the dual mode, i.e. one that requires platforms to choose the same mode (either seller or marketplace) across all its products, is more likely to be harmful for consumers and welfare than just banning the dual mode at the level of an individual product or a narrow product category. For this reason, in this paper we study the welfare implications of banning the dual mode for an individual product. If we find a narrow ban on the dual mode (at the product level) is bad for consumers or welfare, then a wider ban that prevents the platform acting as a seller for some products and a marketplace on others is even more likely to be harmful.

A number of antitrust concerns have been raised when a dominant platform adopts the dual mode at the product level. These all center on the possibility that the platform may want to favor the products it sells and so distort competition in the marketplace, leading to unfair competition.[1] This can happen in at least two important ways. One is that the platform obtains proprietary information on the third-party sellers' products (e.g. detailed demand and pricing data, data on users' search behavior, what items they return and their reasons for doing so) via its marketplace, and then uses that opportunistically to decide whether to copy and compete on the more successful offerings, potentially leading to reduced incentives for third-party sellers to invest or innovate.[2] A second channel is that the platform can steer consumers towards its own offerings (or affiliated products) rather than those offered by third-party sellers by displaying its own offerings more prominently, a practice that has become known as self-preferencing. In Amazon's case this can arise through its Buybox, which around 85% of consumers click on to complete their order. This allocates a seller to the consumer according to a secret algorithm that Amazon controls, and oftentimes the allocated seller is Amazon itself.

To model these practices, our analysis features a platform M that can function as a seller and/or a marketplace, a fringe of small third-party sellers that sell an identical product, and an innovative seller S whose product is superior to all other products. We allow for the possibility that consumers can bypass the platform and purchase directly from third-party sellers, i.e. from the sellers' own websites, from the sellers' own physical stores, or through some alternative channel where the sellers' products are available.[3] Specifically, consumers have heterogenous

[1] In March 2019, U.S. Senator Elizabeth Warren published a policy proposal for curbing the power of big technology firms, which included the following statement: "Many big tech companies own a marketplace, where buyers and sellers transact, while also participating on the marketplace. This can create a conflict of interest that undermines competition. Amazon crushes small companies by copying the goods they sell on the Amazon Marketplace and then selling its own branded version." See https://medium.com/@teamwarren/heres-how-we-can-break-up-big-tech-9ad9e0da324c

[2] See Mattioli (2020) for reports that Amazon used data from its own sellers to launch competing products.

[3] With Shopify and other enabling technologies, the direct channel is becoming an increasingly important option, even for purely online sellers. From 2016 to 2019, direct-to-consumer (DTC) ecommerce grew at an estimated three to six times the rate of overall ecommerce sales in the U.S., with the 2019 level being 14.28 billion dollars (eMarketer, April 2, 2020).

preference for transaction channels: some consumers obtain a convenience benefit from purchasing through the platform and they complete transactions through whichever channel offers them the highest utility (we call them "regular consumers"), whereas others have an overriding preference to transact with sellers directly (we call them "direct consumers"). In the case of products that are also sold through physical stores, one can think of direct consumers as those who stick to buying offline. We explore three different business models for the platform: pure marketplace (facilitating transactions by third-party sellers who set their own prices for these transactions), pure seller (selling products in competition with outside sellers), and dual (operating in both modes, selling in its own name at the same time as facilitating transactions by third-party sellers).

Our first finding is that the dual mode is the most profitable for the platform. By hosting S on the marketplace, the platform avoids head-to-head cross-channel competition with S's superior product, while at the same time extracting some surplus from S's product through its transaction commission. The relaxation of head-to-head cross-channel competition from hosting S comes from the fact S has two price instruments (its marketplace price and its direct price), allowing it to price discriminate between regular consumers through its marketplace price and direct consumers through its direct price. Even though the pure marketplace mode also offers the benefit of avoiding head-to-head competition, in dual mode the presence of M's offering constrains S's price on the marketplace, so that for any given commission level consumers are more likely to purchase through the marketplace. This margin squeeze on S allows M to raise its commission in dual mode above the commission it optimally charges in the pure marketplace mode without causing consumers to prefer to buy directly from S.

We then analyze the effect of a ban on the dual mode, taking into account that M endogenously decides which of the two pure modes to switch to in response to the ban. If S's product is sufficiently good, M switches to operate as a pure marketplace. This results in higher profits for S, at the expense of M and consumer surplus, while total welfare remains unchanged. This is because S is no longer constrained by competition with M's product, so S can set a higher price in both channels to fully extract the additional surplus that its superior product offers to consumers.

If instead S's product is not much better than M's, a ban on the dual mode results in M switching to operate as a pure seller. This results in lower total welfare because the dual mode gives consumers the option to purchase S's superior product through M's more convenient channel, whereas this option is unavailable in seller mode. However, the effect on consumers is, in general, ambiguous and reflects two opposing forces. On the one hand, in the pure seller mode the head-to-head cross-channel competition for regular consumers means S sets a low price in the direct channel, resulting in a benefit to direct consumers. On the other hand, regular consumers become worse-off because they can no longer enjoy the convenience benefit of buying S's product via M.

We then use our framework to explore the practices of product imitation and self-preferencing that have raised antitrust scrutiny. To do so, we modify our baseline model by assuming: (i) S endogenously chooses the innovation level and M can imitate S's innovative product whenever the product is hosted on M's marketplace; and (ii) regular consumers rely on M's recommendation to discover S's innovative new product, and they are otherwise unaware of S (including S's direct channel). There are three key implications from these modifications. First, expecting M to imitate its product, S has a weaker incentive to innovate in the dual mode compared to the marketplace mode. Second, upon imitating S in dual mode, M's imitation product offers the highest net surplus, meaning that M earns the highest profit by selling its product rather than by collecting commissions. As a result, in equilibrium M always engages in self-preferencing. Third, M's ability to steer consumers towards its own product essentially eliminates the price competition with S's product, allowing M to charge a high price for its imitation product.

We consider four possible policy interventions to address the harm arising from product imitation and self-preferencing. First, we show that outright banning the dual mode turns out to be ineffective, because it results in M choosing the seller mode (so that S is unable to sell to regular consumers given it would no longer get discovered by them), while charging a relatively high price for the product it sells. As such, banning the dual mode does not restore S's incentive to innovate or result in stronger price competition. The ban lowers total welfare without improving consumer surplus or S's profit.

Second, we show that banning product imitation alone restores S's innovation incentive because in the post-intervention equilibrium M continues to operate in dual mode and S sells to all consumers. At the same time, the ban prohibits the innovative product from being combined with M's cost advantage. We show that the improvement in innovation level dominates the cost saving (so that welfare increases with the ban) if and only if S's innovation is sufficiently cost-efficient (in a sense we will make precise). Nonetheless, M's ability to steer consumers still allows it to charge a high commission in dual mode, which in turn results in a high price on the platform. Thus, banning imitation alone may improve welfare but does not improve consumer surplus.

Third, we show that banning self-preferencing alone (i.e. requiring M to always recommend the product that offers the highest surplus for consumers) restores price competition, and M still operates in dual mode in the post-intervention equilibrium. Specifically, the ban means that regular consumers are effectively exposed to all offers available, which intensifies the competition between M and S and lowers the final price paid by consumers. On the other hand, the ban does not restore S's innovation incentive because M continues to imitate S's product, implying that total welfare remains unchanged.

Finally, we consider banning both product imitation and self-preferencing, which can result in M either choosing the seller mode or continuing in dual mode. The latter is true whenever M's cost advantage is sufficiently small: in this case, the policy leads to higher consumer surplus and profit for S. In addition, welfare improves if and only if S's innovation is sufficiently cost-efficient, capturing the same trade-off as in the case of banning product imitation alone. These results reflect that banning product imitation and self-preferencing address the negative consequences of each of these practices, while at the same time preserving the benefits of the dual mode captured in our baseline model. Comparing the implications of all four policy interventions, our results suggest that a structural ban on the dual mode is a less effective intervention than behavioural remedies.

The rest of the paper proceeds as follows. In Section 1.1 we survey the related literature.

We lay out the baseline model in Section 2 and analyze it in Section 3, where we compare the three modes that the platform can choose and the implications of banning the dual mode. In Section 4 we introduce product imitation and self-preferencing by the platform, and compare the four policy interventions mentioned above: banning the dual mode, banning imitation alone, banning self-preferencing alone, and banning both imitation and self-preferencing. Section 5 explores several extensions of our framework: exploring what happens when there are multiple product categories, allowing for competing platforms that can endogenously choose the mode of their operations, and comparing the marketplace-seller dual mode analyzed here to the more traditional case in which the intermediary is a retailer (like a supermarket) that can offer its own in-house brands alongside products sourced from third-party suppliers. Finally, in Section 6 we conclude.

1.1 Related literature

A recent strand of literature has emerged that compares the platform business model with various alternative models: marketplace or reseller (Hagiu and Wright, 2015a), platform or vertically integrated firm (Hagiu and Wright, 2015b and 2018), agency or wholesale pricing (Johnson, 2017). In these papers, the key distinction between the business models is the delegation of control rights over key factors that are relevant for total demand, e.g. prices and marketing choices. This literature does not consider the possibility of the dual mode, in which a platform operates a marketplace and acts as a seller itself (competing with third-party sellers) on its marketplace.

Somewhat closer is the literature that considers whether a platform should offer its own products or services.[4] For example, Hagiu and Spulber (2013) consider a platform facing the chicken-and-egg coordination problem in user participation, showing that this problem can be mitigated by introducing first-party content alongside third-party content. Farrell and Katz (2000) and Jiang, Jerath, and Srinivasan (2011) analyze platform owners that face a tradeoff between extracting rents and motivating innovation by third-party complementors. Zhu and Liu (2018) empirically investigate this question, showing that Amazon is more likely to compete with its marketplace sellers in product categories that are more successful in terms of sales. A distinction relative to our paper is that this literature assumes that all products or services have to be sold through the platform (i.e. there is no direct channel) and does not consider the surplus and welfare implications of the dual mode.

Our analysis of banning the dual mode when there is self-preferencing (Section 4) is related to the analysis by de Cornière and Taylor (2019), which considers a vertically-integrated intermediary that biases its recommendations in favor of its subsidiary seller at the expense of third-party sellers.[5] Divestiture (which would eliminate the dual role played by the intermediary) means both the intermediary and the seller coexisting and operating independently in their setup. Among several results, they show that divestiture can increase consumer surplus under price competition.[6] Our analysis of self-preferencing differs in many respects, including that

[4]Hagiu et al. (2020) consider the opposite situation of a traditional firm hosting rivals to become a platform.

[5]See also the first section of Calvano and Polo (forthcoming) for a comprehensive survey on the economic literature of biased intermediation by digital platforms.

[6]See also Gilbert (2020) and Krämer and Zierke (2020) on how vertical separation can sometimes decrease

we also allow for imitation, that we endogenize the intermediary's post-ban choice of business model, and that we allow for multiple channels (direct vs. intermediated).

An important part of the mechanism driving our results rests on the ability of (regular) consumers to choose which channel to buy from, i.e. through the platform or directly. This relates to some recent work that explicitly models the direct purchase channel option, e.g. Edelman and Wright (2015), Wang and Wright (2020) and Ronayne and Taylor (2019) among others, but none of these papers consider a platform's ability to operate in dual mode.

In our paper, a platform that operates in dual mode can be viewed as a vertically integrated firm that uses the upstream input (the facilitation of transactions through the marketplace) to offer downstream products (selling its product through the marketplace) that compete with other downstream sellers. The literature on vertical foreclosure has studied how upstream market power leads to negative effects on downstream competition (e.g. Rey and Tirole, 2007). Our setting is different in several respects, including importantly that the platform is not an essential facility in our setting, so third-party sellers can still sell to consumers directly. Similarly, the literature on access pricing has explored how a vertically integrated incumbent distorts its competition with downstream firms through inefficient demand-sabotage (e.g. Mandy and Sappington, 2007; Brito et al., 2012). While demand-sabotage is analogous to self-preferencing under the dual mode in our analysis, one key conceptual difference is that self-preferencing does not directly reduce efficiency because consumers still purchase the product with the highest net value on the equilibrium path. Rather, self-preferencing generates an inefficiency only through its effect on third-party's incentive to innovate rather than any direct inefficiency associated with an actual act of sabotage.

Finally, concurrent with this paper, Etro (2020) also studies the issue of platforms like Amazon that can decide for any product whether buyers purchase from itself or from a third-party seller. The paper addresses Amazon's incentives to enter with private label products or as a reseller of third-party products, and shows its incentive to enter aligns with consumers' interests when sellers are perfectly competitive, but there is generally insufficient entry by Amazon when sellers have market power. We differ in that we focus on the strategic price competition between the intermediary and the third-party seller (both on-platform and cross-channel) and that we endogenize the intermediary's post-ban choice of business model, both of which lead to significant differences in how a ban on the dual mode works.

2 Model setup

Suppose each consumer wants to buy one unit of one product where there is a continuum (measure one) of consumers. Transactions can be performed directly or through a platform (or more generally, an intermediary) M.

The product is supplied by $n \geq 2$ identical "fringe sellers", and the products are each valued at v by consumers. In addition, there is a superior seller S which benefits from an existing innovation, such that its product is valued at $v + \Delta > v$. The marginal costs of S and fringe sellers are constant at $c \geq 0$. Depending on the mode of operation, M may be able to operate

surplus and efficiency in similar settings.

as a seller and sell its own offering to consumers. In this case, M's offering may be an existing product sourced from competitive manufacturers or a product produced and sold under M's own brand. We do not distinguish between these two possibilities in the model, which is equivalent to assuming that manufacturers, if they exist, behave non-strategically. This implies the "seller" mode covers both cases in which the platform sells products sourced from manufacturers (e.g. Amazon, Target and Walmart acting as retailers of other brands' products) and cases in which the platform sells first-party products that it either makes itself or contract manufactures under its own brand (e.g. Apple or Google's own apps, or AmazonBasics).[7]

In the baseline version of the model we assume M cannot copy S's product, and like the fringe firms, its product is also valued at v. However, due to economies of scale, M has a cost advantage over sellers and has potentially a lower marginal cost, which we normalize to zero so that the parameter c captures the relative cost efficiency of M. Setting $c = 0$ captures the case when M has no cost advantage over the other sellers.

There are two types of consumers. They differ in the benefit they get from using M to complete transactions. A fraction $0 < \mu < 1$ of consumers dislike using M to make transactions. Specifically, we assume these consumers face a sufficiently large inconvenience (or transaction cost) of using M for a transaction, so using M is never a relevant consideration for them.[8] We call them direct consumers: we have in mind that they always buy directly from the sellers (the sellers' own websites) or from alternative channels in which the sellers' products are available (e.g. from other platforms or retail stores). The remaining fraction $1 - \mu$ of consumers enjoy a convenience benefit $b \geq 0$ of using M to conduct transactions. We call these regular consumers. In Section B of the Online Appendix we extend our benchmark analysis to allow for a continuum of consumer types that vary in their draws of b.

The existence of some direct consumers is key to ensuring that in dual mode M can sometimes extract more than its transaction benefit b in fees without causing all consumers to buy from S. The basic idea is that because of direct consumers, S will sometimes prefer to exploit these consumers rather than trying to induce regular consumers to buy outside by lowering its price in the direct channel to compete with the product sold by M. Note we do not require a large number of direct consumers for our findings to hold. Indeed, our results go through in the limit when $\mu \to 0$. Without any direct consumers ($\mu = 0$), M would be indifferent between the marketplace mode and the dual mode, and so the question of whether to ban the dual mode would not arise if M breaks the tie in favor of operating in marketplace mode. However, provided M always breaks the tie in favor of operating in dual mode, our analysis continues to hold.

Consumers always have an outside option of not buying anything, which gives zero value. We assume $v > c$ so consumers always buy something. We also assume $\Delta > c$ so that S's product innovation is a more important dimension than M's cost efficiency, as otherwise in dual mode there can be no equilibrium where S makes any sales via M, and the dual mode

[7] In Section A of the Online Appendix, which is available at https://www.wrighteconomics.com, we consider what happens if M can instead source its product from S (so S sells its product to M in the wholesale market). There we show the main implications of banning the dual mode remain the same.

[8] A sufficient condition is that the inconvenience cost these consumers face from using M for a transaction exceeds c (in the baseline model) or Δ (in the extended model of Section 4). Alternatively, it could be that these consumers are unaware of M's existence.

simply reduces to the seller mode. Finally, we assume that one of the two parameters b and c is positive to exclude the uninteresting case where M's profit is the same in all modes because M can never charge a positive price and attract any demand in any of the modes since it doesn't offer any efficiency benefit.

Whenever M's mode includes a marketplace, it charges a commission τ to third-party sellers for each transaction facilitated. Third-party sellers (including S) can choose whether to participate on M's marketplace, and whenever they do, can price discriminate between consumers that come to it through the marketplace and consumers that come to it through the direct channel. We posit that all third-party sellers always participate on M's marketplace if they are indifferent. Given there are always two or more identical fringe sellers competing in the direct channel and on the marketplace, following the standard Bertrand logic, we take as given that fringe sellers always price at marginal cost, i.e. c if selling directly and $c + \tau$ if selling on a marketplace, regardless of how S and M price. Thus, when we characterize equilibria, we take these fringe seller prices as given.[9]

Throughout, we solve for subgame perfect Nash equilibria. To ensure equilibria are well defined, we assume that consumers break ties in favor of S's product whenever they are indifferent between multiple products including S's product, and then break ties in favor of the product sold by M whenever they are indifferent between that and the fringe sellers' products. Note these tie-breaking rules do not apply when regular consumers are indifferent between S's products sold in different channels, in which case we state the tie-breaking rule as part of the equilibrium construction.

Finally, whenever there are multiple equilibria in any subgame that are payoff-equivalent for S, but payoff-ranked by M, we select the one preferred by M, and similarly, whenever there are multiple equilibria in any subgame that are payoff-equivalent for M, but payoff-ranked by S, we select the one preferred by S.

3 Banning dual mode in the baseline model

In this section we characterize the equilibria arising after M's choice of each of the three possible modes: marketplace mode, seller mode, or dual mode, as well as the consequences of banning the dual mode. In period zero, M chooses one of these modes, and this becomes common knowledge. Implicit in this timing assumption is that M can commit to its choice of mode.[10] The timing for subsequent periods is specified for each mode in what follows.

3.1 Marketplace mode

Suppose M chooses the marketplace mode. **Timing**: (1) M sets its commission $\tau \geq 0$ to be paid by sellers on each sale made through the marketplace; (2) S chooses whether to participate;

[9] Thus, throughout the paper, we rule out equilibria supported by fringe suppliers pricing below cost, i.e. playing weakly dominated strategies.

[10] In practice, this commitmnent could be reinforced by building a reputation for sticking to a particular mode. For example, the e-commerce platform eBay has remained a pure marketplace since its inception. SAP regularly publishes product roadmaps for its Netweaver platform in order to let third-party application developers know which products the company will sell and which products it will not.

(3) sellers set prices simultaneously, both inside (on the marketplace) and outside (when selling directly to consumers).

Suppose first $\tau \leq b$. If S has joined the marketplace, then S's price on the marketplace ("inside" price) p_i and direct ("outside") price p_o are bounded above by the fringe sellers' prices after adjusting for the superior quality of S's product, i.e. $p_i \leq c + \tau + \Delta$ and $p_o \leq c + \Delta$. Clearly, given our tie breaking rule, S does best by setting its inside and outside prices so these constraints bind, implying $p_i^* = c + \Delta + \tau$, $p_o^* = c + \Delta$, and it earns a post-participation profit of Δ. There is no incentive for S to deviate by inducing regular consumers to switch to buy from the direct channel because doing so requires it to lower direct price to $p_o \leq c + \Delta + \tau - b$, with deviation profit that is below Δ given $\tau \leq b$. If S has not joined the marketplace, then it has to compete with fringe sellers listed on M as well as fringe sellers at the outside channel. In this case, S either optimally sets $p_o^* = c + \Delta$ and sells only to direct consumers (profit is $\mu\Delta$) or sets $p_o^* = c + \Delta + \tau - b$ to sell to all consumers (profit $\Delta + \tau - b$). For both possibilities, S's non-participation profit is weakly lower than Δ, and so it always joins M.

If $\tau > b$, then the logic above implies that M earns zero profit. Indeed, in this case, regardless of whether it participates or not, S sets its direct price so that it induces all regular consumers to buy outside in the equilibrium of the pricing subgame.

Given the equilibrium in the pricing subgames characterized above, we have:

Proposition 1 *(Marketplace mode) In the equilibrium, M sets $\tau^{market} = b$, S sets $p_i^* = c+\Delta+b$ and $p_o^* = c+\Delta$. All regular consumers purchase from S through the marketplace. The equilibrium profits of M and S are $\Pi^{market} = b(1-\mu)$ and $\pi^{market} = \Delta$ respectively.*

Note in particular that M obtains the same profit regardless of the level of Δ. The reason is that S can fully extract the value of its innovation (inside and outside the platform). Meanwhile, M only extracts surplus up to the transaction benefit it offers.

3.2 Seller mode

Suppose M chooses the seller mode and sets a price p_m for the product it sells. In this case, S only has one price to set: p_o, the outside price. **Timing**: (1) M and all third-party sellers set prices simultaneously.

There are two possible types of equilibria in the seller mode depending on whether M makes any sales. First, if Δ is large, there is a unique pure-strategy equilibrium in which S makes all the sales, with $p_o^* = \Delta - b$ and $p_m^* = 0$. Here S sells to all consumers and earns $\pi^{sell} = \Delta - b - c$, while M earns $\Pi^{sell} = 0$. In this case, S has no incentive to deviate to $p_o = c + \Delta$ to exploit the direct consumers, as long as $\Delta \geq \frac{b+c}{1-\mu}$. On the other hand, when $\Delta < \frac{b+c}{1-\mu}$, S would have an incentive to deviate from this pure-strategy equilibrium by setting $p_o = c + \Delta$ to exploit direct consumers. In this case, given that direct consumers are essentially "captive" to S, the usual logic of the Bertrand-Edgeworth cycle implies that there can be no pure-strategy equilibrium, and the equilibrium has to be in mixed-strategies in which M sometimes makes a positive amount of sales. In the special case of $\Delta = \frac{b+c}{1-\mu}$, these two types of equilibria co-exist and our selection rule implies the equilibrium with M earning a positive profit is selected. Formally,

Proposition 2 *(Seller mode)*

- *If* $\Delta > \frac{b+c}{1-\mu}$, *in equilibrium* $p_o^* = \Delta - b$ *and* $p_m^* = 0$. *All regular consumers purchase from* S *directly. Equilibrium profits are* $\Pi^{sell} = 0$ *and* $\pi^{sell} = \Delta - b - c$.

- *If* $\Delta \leq \frac{b+c}{1-\mu}$, *in the mixed-strategy equilibrium,* p_m^* *is distributed according to c.d.f* F_m *with support* $[c + b - (1-\mu)\Delta, c + b]$, *where*

$$F_m(p_m^*) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b + \Delta - c}\right) \text{ for } p_m^* \in [c + b - (1-\mu)\Delta, c + b], \quad (1)$$

and p_o^* *is distributed according to c.d.f* F_o *with support* $[c + \mu\Delta, c + \Delta]$, *where*

$$F_o(p_o^*) = \begin{cases} 1 - \left(\frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta}\right) & \text{for } \; p_o^* \in [c+\mu\Delta, c+\Delta) \\ 1 & \text{for } \; p_o^* \geq c + \Delta \end{cases}. \quad (2)$$

Equilibrium profits are $\Pi^{sell} = (c + b - (1-\mu)\Delta)(1-\mu)$ *and* $\pi^{sell} = \mu\Delta$. *This is the unique equilibrium when* $\Delta < \frac{b+c}{1-\mu}$.

Here, $\Delta - \frac{b+c}{1-\mu}$ captures S's incentive to dominate the market (i.e. selling to all consumers). For $\Delta > \frac{b+c}{1-\mu}$, the overall efficiency of S's product is much higher than M's and the mass of regular consumers is sufficiently large, so that the pricing equilibrium involves S taking the whole market rather than exploiting direct consumers. All regular consumers buy from S, resulting in M earning zero profit. For $\Delta \leq \frac{b+c}{1-\mu}$, either (i) the efficiency of S's product is too small or (ii) the mass of regular consumers is too small, so that S is unwilling to serve the whole market at a low price, and so both M and S make a positive amount of sales to regular consumers in the mixed strategy equilibrium.

Consider the extreme case in which $\mu \to 0$, so the condition for the mixed strategy equilibrium to prevail converges to $\Delta \leq b + c$. When $\mu \to 0$, the c.d.f. F_m has all its mass concentrated at the lower bound $c + b - \Delta$. In this case, M's offering has a higher overall efficiency so it takes the whole market, and S's profit converges to zero.[11] At the other extreme, if $\mu \to 1$, then S chooses to exploit direct consumers by pricing with a distribution that converges to the price $c + \Delta$, and M competes to attract regular consumers by pricing with a distribution that converges to the price $c + b$, with M's profit converging to zero.

3.3 Dual mode

Now suppose M acts as both a marketplace and a seller, meaning regular consumers can choose between buying from M (as a seller) and buying from third-party sellers who participate on M's marketplace. **Timing**: (1) M sets the marketplace commission τ; (2) S chooses whether or not to participate; (3) All sellers, including S and M, set prices simultaneously.

[11] When μ is exactly zero, there is also a pure-strategy equilibrium: the standard asymmetric Bertrand outcome, in which $p_m^* = c + b - \Delta$ and $p_o^* = c$, along with the mixed-strategy equilibrium.

3.3.1 Post-participation pricing subgame

We solve by backwards induction, considering first the stage-3 pricing subgame in which S participates on M's marketplace. There are three possible types of equilibria in this subgame, depending on the purchase decisions of regular consumers:

- *Marketplace* equilibrium: all regular consumers buy from S through the marketplace.
- *Direct sales* equilibrium: all regular consumers buy from S directly.
- *Seller* equilibrium: a positive fraction of regular consumers buy from M and the equilibrium can be in mixed strategies.

First, consider the marketplace equilibrium in which regular consumers buy from S through the marketplace. Then S's equilibrium inside price satisfies $p_i^* \leq \tau + \Delta$, because otherwise M could earn a higher margin than τ by undercutting p_i^* and selling its offering. Given that the marginal costs of S and M are $c+\tau$ and 0 respectively, in principle any inside-price profile $p_i^* \in [\max\{c+\tau, \Delta\}, \tau+\Delta]$ and $p_m^* = p_i^* - \Delta$, in which S sells to all regular consumers, is a possible candidate equilibrium.[12] Even though M's marginal cost is zero, it does not necessarily want to undercut S by setting $p_m^* < p_i^* - \Delta$ because $p_i^* \leq \tau + \Delta$ implies that undercutting leads to a smaller margin than the commission τ collected.[13] For each candidate equilibrium it remains to check whether S can profitably deviate to induce regular consumers to the outside channel. Naturally, such a deviation is not profitable as long as τ is not too high.

Lemma 1 *(Dual mode, marketplace) Fix τ and define $\hat{p} = c + \Delta + \frac{b-\tau(1-\mu)}{\mu}$. Any price profile satisfying $p_i^* \in [\max\{c+\tau, \Delta\}, \min\{\hat{p}, \tau+\Delta\}]$, $p_m^* = p_i^* - \Delta$, and $p_o^* = c + \Delta$ is a marketplace equilibrium. The marketplace equilibria exist if and only if $\tau \leq b + \mu \min\left\{\frac{b+c}{1-\mu}, \Delta\right\}$. The equilibrium profits are $\Pi = \tau(1-\mu)$ and $\pi = \mu\Delta + (1-\mu)(p_i^* - c - \tau)$.*

Next consider the direct sales equilibrium. In such an equilibrium, M earns zero profit because all sales occur directly. Given this, it will undercut and attract regular consumers back to the marketplace as long as $p_o^* > \Delta - b$. Consequently, such equilibria necessarily have $p_o^* = \Delta - b$, $p_m^* = 0$ and $p_i^* > \Delta$, and S earns $\Delta - b - c$ in equilibrium. It remains to check whether S has an incentive to deviate to $p_o = c + \Delta$ to fully exploit the direct consumers (earning profit $\mu\Delta$) instead of setting the low equilibrium price that attracts all consumers to the direct channel. Comparing S's equilibrium profit $\Delta - b - c$ against the relevant deviation profit, such a deviation is profitable only if Δ or τ is small:

Lemma 2 *(Dual mode, direct sales)*

[12] Strictly speaking, there can be other equilibria involving $p_m < 0$. However, for any given τ these equilibria involve M playing weakly dominated strategies. We rule out such equilibria because they involve M setting a price that it would prefer to change if some consumers actually purchased from it (i.e. off the equilibrium path).

[13] This does not mean $p_m < \tau$ is weakly dominated. This is because in cases where τ is high and p_o is low so that consumers buy from S directly, M earns zero commission and it can profitably set a price below τ to attract regular consumers.

- *If* $\Delta \geq \frac{b+c}{1-\mu}$ *and* $\tau \geq \frac{b+c\mu}{1-\mu}$, *then any price profile satisfying* $p_i^* > \Delta$, $p_m^* = 0$, *and* $p_o^* = \Delta - b$ *is a direct sales equilibrium. The equilibrium profits are* $\Pi = 0$ *and* $\pi = \Delta - b - c$.

- *If* $\Delta < \frac{b+c}{1-\mu}$ *or* $\tau < \frac{b+c\mu}{1-\mu}$, *then there is no direct sales equilibrium.*

The derivation of the seller equilibrium follows the case in which M operated as a pure seller, but with two important differences: (i) if M wins the inside competition and makes any sales, its price p_m is capped by a tighter constraint $\min\{c+\tau-\Delta, c+b\}$; (ii) M can sometimes let S win the inside competition by setting $p_m = p_i - \Delta$ to earn commissions instead of making sales by itself, and this strategy is feasible as long as τ is not too high so that in equilibrium S is still willing to make sales inside. However, if τ is sufficiently high, S's inside product is irrelevant, and the outcome is the same as the seller mode.

Lemma 3 *(Dual mode, seller)*

- *If* $\Delta > \frac{b+c}{1-\mu}$ *or* $\tau < b + \mu\Delta$, *then there is no seller equilibrium.*

- *If* $\Delta \leq \frac{b+c}{1-\mu}$ *and* $\tau \in [b+\mu\Delta, b+\Delta)$, *then in the mixed-strategy equilibrium,* $p_i^* = c+\tau$; p_m^* *is distributed according to c.d.f* F_m *with support* $[c+b-(1-\mu)\Delta, c+\tau-\Delta]$, *where*

$$F_m(p_m^*) = \begin{cases} \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b + \Delta - c}\right) & \text{for } \; p_m^* \in [c+b-(1-\mu)\Delta, c+\tau-\Delta) \\ 1 & \text{for } \; p_m^* \geq c+\tau-\Delta \end{cases};$$

p_o^* *is distributed according to c.d.f* F_o *with support* $[\mu\Delta + c, c+\tau-b] \cup \{c+\Delta\}$, *where*

$$F_o(p_o^*) = \begin{cases} 1 - \frac{c+b-(1-\mu)\Delta}{p_o^* - \Delta + b} & \text{for } \; p_o^* \in [\mu\Delta + c, c+\tau-b) \\ 1 - \frac{c+b-(1-\mu)\Delta}{\tau} & \text{for } \; p_o^* \in [c+\tau-b, c+\Delta) \\ 1 & \text{for } \; p_o^* \geq c+\Delta \end{cases}.$$

In equilibrium S makes some inside sales. Regular consumers buy from S directly when they are indifferent between S's product in multiple channels. The equilibrium profits are $\Pi = (b+c-(1-\mu)\Delta)(1-\mu)$ *and* $\pi = \mu\Delta$.

- *If* $\Delta \leq \frac{b+c}{1-\mu}$ *and* $\tau \geq b+\Delta$, *then any price profile satisfying* $p_i^* > c+b+\Delta$, *with* p_o^* *and* p_m^* *being distributed according to c.d.f (1) and (2), is a mixed-strategy seller equilibrium. In equilibrium S makes no inside sales, and the equilibrium profits are* $\Pi = (b+c-(1-\mu)\Delta)(1-\mu)$ *and* $\pi = \mu\Delta$.

To summarize the stage-3 equilibria in the subgame with S's participation, in the table below we list down the relevant range of parameters and state the corresponding equilibria that exist:

- In the marketplace equilibria (ME), $\Pi = \tau(1-\mu)$ and $\pi = \mu\Delta + (1-\mu)\min\left\{\Delta + \frac{b-\tau}{\mu}, \Delta - c\right\}$

- In the direct sales equilibria (DE), $\Pi = 0$ and $\pi = \Delta - b - c$.

- In the seller equilibria (*SE*), $\Pi = (b + c - (1-\mu)\,\Delta)\,(1-\mu)$ and $\pi = \mu\Delta$.

	$\tau < b + \mu\Delta$	$\tau = b + \mu\Delta$	$\tau > b + \mu\Delta$
$\Delta < \frac{b+c}{1-\mu}$	*ME*	*ME,SE**	*SE*
$\Delta = \frac{b+c}{1-\mu}$	*ME*	*ME,DE,SE**	*DE,SE**

	$\tau < \frac{b+c\mu}{1-\mu}$	$\tau = \frac{b+c\mu}{1-\mu}$	$\tau > \frac{b+c\mu}{1-\mu}$
$\Delta > \frac{b+c}{1-\mu}$	*ME*	*ME,DE**	*DE*

* π is the same across all coexisting equilibria

(3)

Notice that whenever at least two out of the three types of equilibria coexist, S's profit does not vary across co-existing equilibria. Therefore, based on our selection rule, we select the marketplace equilibrium whenever it is available, which is the best for M, followed by the seller equilibrium and the direct sales equilibrium.

3.3.2 Overall equilibrium

If S does not participate, the pricing subgame unfolds as if M operates as a pure seller, except that its price is bounded by $p_m^* \leq c + \min\{\tau, b\}$ due to the existence of fringe sellers both on the marketplace and outside. The equilibrium characterization in this case is largely similar to Proposition 2: if Δ is large, S sells to all regular consumers; if Δ is small, M sells to some regular consumers. In the proof of the next proposition, we show that in the equilibrium of the subgame following non-participation, S's profit is $\pi_{np} = \max\{\mu\Delta, \Delta - b - c\}$. Compared to the post-participation profit described by Table 3, it follows that S always weakly prefers to participate.

We can now derive the overall equilibrium (including M's choice of τ in stage 1):

Proposition 3 *(Dual mode equilibrium) M sets $\tau^{dual} = b + \mu \min\left\{\frac{b+c}{1-\mu}, \Delta\right\}$ and S participates. In the resulting marketplace equilibrium:*

- *If $\Delta > \frac{b+c}{1-\mu}$, the equilibrium prices are $p_o^* = c + \Delta$, $p_i^* = \Delta$, and $p_m^* = 0$.*
- *If $\Delta \leq \frac{b+c}{1-\mu}$, the equilibrium prices are $p_o^* = c + \Delta$, $p_i^* = c + \tau^{dual}$, and $p_m^* = c + \tau^{dual} - \Delta$.*

All regular consumers buy from S on M and direct consumers buy directly. Equilibrium profits are $\Pi^{dual} = \tau^{dual}\,(1-\mu)$ and $\pi^{dual} = \max\{\Delta - b - c, \mu\Delta\}$.

In this equilibrium, the competition with M on the marketplace effectively imposes a cap on S's inside price. By combining this cap with a higher commission, M can squeeze S's margin while still maintaining S's incentive to sell through its marketplace. When $\Delta > \frac{b+c}{1-\mu}$, the margin squeeze is only partial in the sense that S still earns a positive margin. Intuitively, a high Δ means that if M wants to cap S's inside price, then M would need to set $p_m < 0$. However, such pricing below marginal cost is a weakly dominated strategy in the pricing subgame, and so the tightest possible price cap M can impose on S (achieved when $p_m = 0$) is $p_i^* = \Delta > c + \tau^{dual}$. When $\Delta \leq \frac{b+c}{1-\mu}$, M can cap S's inside price at $p_i^* = c + \tau^{dual}$ while still setting a non-negative p_m.

The mechanism of margin squeeze in dual mode relies on $\mu > 0$. If $\mu = 0$ and $\tau > b$, then in any candidate equilibrium with S selling inside, S can deviate by lowering its direct price, attracting regular consumers to the direct channel, and hence avoiding having to pay a commission. Reflecting this logic, when $\mu \to 0$ we have $\tau^{dual} \to b = \tau^{market}$ and so $\Pi^{dual} \to \Pi^{market}$. Having $\mu > 0$ creates an opportunity cost for S to lower its direct price, thus allowing M to set $\tau > b$ while still having S sell inside.

3.4 Comparisons of the different modes

Before determining which mode M prefers, there is one other mode that we need to rule out first. Specifically, M could choose to own independent marketplace and seller divisions, but with separate teams that are not allowed to communicate or coordinate with each other (i.e. "functional separation" in the terminology of Gilbert, 2020). As we show in Section C in the Online Appendix, in this case, its separate seller division would earn zero profits and so has no incentive to keep operating. Indeed, taking into account any, arbitrarily small, fixed costs that the seller division would face to operate, we can rule out this outcome as part of any equilibrium.[14]

We first compare the equilibrium profits of M and S across the three modes.

Proposition 4 *(Profits comparison)*

- *M's profit: $\Pi^{dual} > \max\left\{\Pi^{market}, \Pi^{sell}\right\}$; Moreover, $\Pi^{market} \geq \Pi^{sell}$ if and only if $\Delta \geq \frac{c}{1-\mu}$, with strict inequality if $b > 0$ and $\Delta > \frac{c}{1-\mu}$.*
- *S's profit: $\pi^{market} > \pi^{dual} = \pi^{sell}$.*

Comparing first across the two pure modes, we see that the marketplace mode is better for M if (i) M's cost efficiency c is small relative to S's innovation surplus Δ, and (ii) the mass of regular consumers is large so that S prices aggressively when competing against M as the seller given it cannot earn much profit from exploiting direct consumers. When these conditions hold, M is better off operating as a marketplace to avoid head-to-head competition with S.

Compared to the marketplace mode, the dual mode allows M to charge a higher commission τ and still have S sell through its marketplace to regular consumers (indeed, $\tau^{dual} > b = \tau^{market}$). To see the intuition for this result, note that in both the dual mode and the marketplace mode, M's commission is constrained by the possibility that S's outside price induces regular consumers to purchase from S directly. However, for any given τ, the inside price is lower in the dual mode than in the marketplace mode due to competition with M, which means it is more difficult for S to shift regular consumers from the marketplace to the direct channel.

[14]In Section C of the Online Appendix we also consider the scenario in which M, facing a ban on the dual mode, can however commit to operate the two separate divisions (for example, because this can sometimes allow it to extract more in its marketplace division, and it can use some of its profit from its marketplace division to cover the fixed costs of operating its seller division).

This in turn relaxes the constraint on M's commission, allowing it to be higher in dual mode. Notably, this conclusion can still hold even if $c = 0$ (i.e. when M has no cost advantage in dual mode). And $\tau^{dual} > \tau^{market}$ immediately implies that $\Pi^{dual} > \Pi^{market}$.

The reason $\Pi^{dual} > \Pi^{sell}$ follows a similar logic. In principle, M can always replicate the outcome of the seller mode by setting a very high τ in the dual mode to deter S from participating. However, in the dual mode the margin squeeze on S's inside sales means M uses its commission to extract some of S's added value over its own offering, i.e. $\Delta - c$. Therefore, M would actually have an incentive to keep S on its marketplace. Reflecting this point, the gap in profit is $\Pi^{dual} - \Pi^{sell} = (1 - \mu)(\Delta - c)$, which is increasing in the added value of S's product.

Comparing the three modes from S's perspective, the marketplace mode is best for S since it can fully extract the value of its innovation on both channels and does not face any direct competition from M. Meanwhile, in order to sustain the proposed equilibrium in the dual mode, the highest fee M can set is such that S is exactly indifferent between selling on M and deviating to sell directly (i.e. compete head-to-head with M). It follows that S earns the same profit in the dual mode and in the seller mode.

Next, consider consumer surplus and welfare, where we distinguish the effects on the surplus of direct consumers and on regular consumers because they can be affected in different ways:

Proposition 5 *(Consumer surplus and welfare).*

- *Welfare:* $W^{dual} = W^{market} \geq W^{sell}$*, where the inequality is strict if* $b > 0$ *or* $\Delta < \frac{b+c}{1-\mu}$*.*
- *Direct consumers:* $CS^{sell}_{\text{direct}} > CS^{dual}_{\text{direct}} = CS^{market}_{\text{direct}}$*.*
- *Regular consumers: If* $\Delta > \frac{b+c}{1-\mu}$*,* $CS^{dual}_{\text{regular}} = CS^{sell}_{\text{regular}} > CS^{market}_{\text{regular}}$*; if* $\Delta \leq \frac{b+c}{1-\mu}$*,* $CS^{dual}_{\text{regular}} > CS^{sell}_{\text{regular}} > CS^{market}_{\text{regular}}$*.*
- *Total consumer surplus: If* $\Delta > b + c$*,* $CS^{sell} > CS^{dual} > CS^{market}$*; if* $\Delta \leq b + c$*,* $CS^{dual} \geq CS^{sell} > CS^{market}$*, where the weak inequality is strict if* $\Delta < b + c$*.*

The welfare result is intuitive. In dual mode, given that S participates and in equilibrium all regular consumers buy from S through the marketplace, the total welfare generated is the same as in the marketplace mode, which is higher than the welfare generated in the seller mode. The inefficiencies in the seller mode reflect (i) the loss in convenience benefits for regular consumers who prefer to buy S through M, and (ii) the fact that in the mixed-strategy equilibrium (which is applicable when $\Delta < \frac{b+c}{1-\mu}$) some regular consumers buy the inferior product sold by M.

The equality $W^{dual} = W^{market}$ obtained here is an artefact of the assumption of discrete consumer types and it does not generally hold when we allow for continuous consumer types based on their draw b of the convenience benefit (or cost) of using M. The analysis of this extension is provided in Section B of the Online Appendix, where we show that the only welfare difference between the dual mode and the marketplace mode is due to a possible distortion arising from cross-channel price differences. To the extent that S's price is lower in one channel than another, this will induce too many consumers to buy in the channel they do not prefer, potentially forgoing their convenience benefit (or cost) from using M. Both modes potentially involve distortions. The marketplace involves the inside price being set higher than the outside price, while in the dual mode this price difference is smaller or even reverses. Provided that the price difference is not reversed in the dual mode, it results in a smaller distortion. Otherwise, the comparison in welfare is ambiguous.

Returning to our setting with discrete consumer types, we find that the marketplace mode is the worst overall for consumers. In this case, S fully extracts the value of its innovation (inside and outside the platform) and M extracts surplus up to the transaction benefit it offers. The other modes leave some of the efficiencies provided by the firms to consumers, and so provide greater consumer surplus. Even though the commission is generally higher in dual mode, the on-platform competition sufficiently suppresses S's inside margin so that the resulting inside price is still lower than in the marketplace mode. Thus, M and consumers benefit from the margin squeeze conducted in the dual mode, at the expense of S.

To compare consumer surplus in the seller mode with the dual mode, note in the dual mode S can price discriminate between regular consumers and direct consumers by charging a low inside price and a high direct price. This implies that direct consumers become worse off while regular consumers become better off compared to the seller mode, with total consumer surplus being higher in dual mode if $\Delta \leq b + c$, but lower in dual mode if $\Delta > b + c$.

3.5 Banning dual mode

A policy that bans the dual mode can result in two possible market structures, depending on whether M chooses to operate in the marketplace mode or the seller mode in period zero. We first compare M's profit in each mode, Π^{sell} and Π^{market}, to determine which mode M would choose after the dual mode is banned. Then, we combine this choice of mode with the surplus comparisons in Proposition 5 to assess the overall impact of a ban on the dual mode.[15]

Proposition 6 *(Ban on dual mode)*

- *If* $\Delta \geq \frac{c}{1-\mu}$*, a ban on the dual mode results in* M *choosing the marketplace mode, with* Π*,* CS_{regular}*, and* CS *decreasing;* π *increasing; and* CS_{direct} *and* W *not changing.*
- *If* $\Delta < \frac{c}{1-\mu}$*, a ban on the dual mode results in* M *choosing the seller mode, with* Π*,* CS_{regular}*, and* W *decreasing;* CS_{direct} *increasing,* π *not changing; and* CS *decreasing if* $\Delta < b + c$*, not changing if* $\Delta = b + c$*, and increasing if* $\Delta > b + c$*.*

Proposition 6 shows that in this benchmark setting, a ban on dual mode results in (weakly) lower welfare, and in most cases, lower consumer surplus. Welfare will be strictly lower whenever M switches to the seller mode after the ban, reflecting the two sources of inefficiencies discussed after Proposition 5. If the proportion of direct consumers μ is small, then $\Delta < \frac{c}{1-\mu}$ implies $\Delta < b + c$, in which case Proposition 6 implies that a ban on dual mode always lowers total consumer surplus. The only way in which banning the dual mode increases total consumer

[15]Note in Proposition 6, the boundary case $\Delta = \frac{c}{1-\mu}$ covers both possibilities since M is indifferent between the two modes, so both of the outcomes listed in the proposition are possible.

surplus is when there is a sufficient proportion of direct consumers, and S has an intermediate level of efficiency advantage $\Delta \in \left(b+c, \frac{c}{1-\mu}\right)$. Intuitively, in the seller mode, the head-to-head competition between M and S means some of the surplus from Δ is left with consumers, but at the same time regular consumers are unable to enjoy the convenience benefits from M facilitating transactions. Whenever $\Delta > b+c$, the gain in surplus from Δ dominates, and so the seller mode leads to higher total consumer surplus. However, if Δ is too high or μ is too low, consumers do not capture this benefit since M prefers to operate as a marketplace. Reflecting this logic, the parameter range for which the ban improves consumer surplus becomes larger when b is low or μ is high, i.e. when the convenience benefit offered by the platform is relatively unimportant.

4 Practices under scrutiny

In this section we explore the two major practices arising from Amazon's use of dual mode that policymakers have been scrutinizing—Amazon imitating third-party products and Amazon steering consumers to the products it sells (self-preferencing). We enrich the baseline model by allowing for the innovation level Δ to be determined by S, and introducing both practices below.[16]

Innovation and product imitation. Suppose that the innovation Δ is chosen by S instead of being exogenously fixed. In this case, M's mode of operation affects S's innovation decision. Let $K(\Delta)$ be the innovation cost for arbitrary innovation level $\Delta \geq 0$, where $K(.)$ is increasing, continuously differentiable, and strictly convex. We define Δ^H and Δ^L such that $K'\left(\Delta^H\right) = 1$ and $K'\left(\Delta^L\right) = \mu$, and assume $\Delta^L > c$ so that $\Delta^H > \Delta^L > c$. These particular innovation levels play a key role in characterizing the equilibrium outcome. We also assume $\mu\Delta^L \geq K(\Delta^L)$ to ensure S is profitable in the equilibrium below. S chooses Δ at the same time as it decides whether to participate in case M operates as a marketplace, and before the price competition stage.

We assume whenever S is available on the platform, M can imitate S's superior product and thereby also offer consumers a product with the additional surplus Δ. This means that we focus on the strongest form of imitation in dual mode—it occurs immediately and works perfectly.[17] If instead S does not participate, then M cannot imitate S's product. This captures the policy concern that by hosting third-party sellers, M obtains proprietary data from them which allows it to copy the relevant product features from that seller.

Product recommendations and steering. In the baseline model, we have assumed that consumers are aware of all available products in the market and then select their preferred purchase channel. However, in practice, many consumers rely on marketplaces like Amazon to find out about the existence of new products. The marketplace, in turn, can manipulate what consumers are able to discover on the marketplace through its recommendation algorithm (i.e.

[16] In Section D of the Online Appendix, we consider what happens when just one practice, imitation or self-preferencing, is introduced. We use this to understand what happens if one of the practices is banned, which we discuss below.

[17] In Section E of the Online Appendix, we allow for imperfect imitation and show that this does not change the main insights.

to steer consumers). To model the issue of platform steering, we assume that regular consumers (mass $1 - \mu$) rely on M's recommendation to find out about products.

Specifically, after all prices are set, M selects a product (or none) to recommend to regular consumers. In addition to the fringe products in the direct channel, which can be interpreted as their *de facto* outside option, regular consumers only know about the product that is recommended by M.[18] Therefore, if M recommends a fringe product or the product it sells (in seller mode or dual mode), regular consumers only choose between the recommended product on M and the fringe products in the direct channel. If instead M recommends S's product, then regular consumers also become aware of its existence in the direct channel and its associated price. They are free to choose which channel to buy S's product from (and still enjoy transaction benefit b if they buy it through M), or they can buy the fringe product directly. Meanwhile, direct consumers (mass μ) behave in the same way as in the baseline model, i.e. they know about all products available in the direct channel and only buy directly.

Timing: (1) M chooses its mode of operation and sets τ (if in marketplace or dual mode); (2) S selects Δ and chooses whether to participate on the platform (if applicable); (3) If M is operating in dual mode and S participates, then M chooses whether to imitate S's product; (4) All sellers, including S and M, set prices simultaneously; (5) M selects a product to recommend to regular consumers.

4.1 Equilibria in each mode

Consider the marketplace mode. We first derive M's profit-maximizing recommendation decision after prices are set. Since the transaction commission τ is uniform, M simply recommends whichever inside product results in transactions on the marketplace, i.e. is preferred by regular consumers over products in the direct channel. Formally, this means M recommends either (i) S if $\Delta + b - p_i \geq \max\{\Delta - p_o, -c\}$ and $b - c - \tau < -c$; or (ii) the fringe product if $\Delta + b - p_i < \max\{\Delta - p_o, -c\}$ and $b - c - \tau \geq -c$. In case (i), M is strictly better off recommending S because otherwise consumers would buy the fringe product directly, while in case (ii), M is strictly better off recommending the fringe product because otherwise consumers would buy S directly. When neither of these cases hold, then M is indifferent between recommendations because consumers either always buy from the marketplace (in which case M always get τ) or buy from the direct channel (in which case M always get zero profit), regardless of M's recommendation. In such a case (which also arises below in the dual mode), we assume that M breaks the tie in favor of recommending the product that provides the highest surplus to consumers.

In the appendix, we show that the recommendation rule above implies the equilibrium characterization of the pricing subgame in the marketplace mode is the same as in the baseline model. In particular, S still extracts the entire innovation surplus and is able to fully pass through the marketplace fee in its pricing. Thus, S's profit function in the innovation stage

[18] Alternatively, we can assume that regular consumers are also unaware of fringe suppliers' products if these products are unavailable on M. As will be clear below, this change does not affect the analysis of the marketplace mode or the dual mode, but it would mean that in the seller mode M can extract the entire regular consumer surplus. This simply shifts the comparisons below in favor of the seller mode from the perspective of M's profit and against the seller mode from the perspective of consumer surplus, without affecting the overall insights.

is $\Delta - K(\Delta)$, which is independent of τ. Hence, it optimally chooses $\Delta^{market} = \Delta^H$. In the overall equilibrium we have $\tau^{market} = b$, $p_i^* = c + \Delta^H + b$, $p_o^* = c + \Delta$, $\Pi^{market} = b(1-\mu)$, and $\pi^{market} = \Delta^H - K(\Delta^H)$.

In seller mode, M necessarily recommends the product it sells to regular consumers. Since these consumers compare M's product only with the fringe sellers' products sold in the direct channel, M can charge up to $p_m^* = c + b$. For any given Δ, in the pricing subgame S sets its outside price as high as possible to extract surplus from direct consumers, subject to the competitive constraint imposed by fringe sellers, so $p_o^* = c + \Delta$ and S's profit is $\mu\Delta - K(\Delta)$. Therefore, S optimally chooses $\Delta^{sell} = \Delta^L$. The equilibrium profits of M and S are $\Pi^{sell} = (b+c)(1-\mu)$ and $\pi^{sell} = \mu\Delta^L - K(\Delta^L)$, respectively.

Now consider the dual mode. We first derive M's profit-maximizing recommendation decision. For any given $\{p_i, p_o, p_m, \tau, \Delta\}$, M recommends S if (i) S's inside product is preferred by regular consumers over the products available in the direct channel (formally, $\Delta + b - p_i \geq \max\{\Delta - p_o, -c\}$) and (ii) the commission is higher than the margin M could earn by trying to sell itself (formally, $p_m \leq \tau$ or $b - p_m \leq -c$). Otherwise, M is better off recommending the product it sells or one of the fringe suppliers' product.

Next, we solve the stage 4 pricing game in case product imitation hasn't occurred in stage 3. For any given τ and Δ, we have:

- If $\tau < b + c$, then $p_m^* = b + c$, $p_o^* = c + \Delta$, p_i^* can be any value, and M recommends the product it sells to all regular consumers.
- If $\tau \in [b + c, b + \Delta]$, then $p_i^* = b + c + \Delta$, $p_o^* = c + \Delta$, p_m^* can be any value, and M recommends S to all regular consumers.
- If $\tau > b+\Delta$, then $p_m^* = b+c$, $p_o^* = c+\Delta$, $p_i^* > b+c+\Delta$, and M recommends the product it sells to all regular consumers.

Thus, M recommends S's product only when $\tau \in [b + c, b + \Delta]$. When $\tau < b + c$, the commission τ is lower than what M can earn from selling itself; when $\tau > b+\Delta$, the commission leads to a high inside price by S, which drives regular consumers to the direct channel. In both of these cases, M prefers to recommend the product it sells.

Now consider the stage 4 pricing game in case product imitation has occurred in stage 3. Given M's inherent cost advantage, it can charge up to $p_m^* = b+c+\Delta$ without inducing regular consumers to buy from their outside option (i.e. buying from fringe suppliers in the direct channel at price c). Hence, for all $\tau \leq b + c + \Delta$, M always recommends its imitation product as the margin is higher than the commission. If instead $\tau > b+c+\Delta$, we know S's inside price is at least $c+\tau > b+2c+\Delta$, which is higher than its highest possible outside price $p_o^* = c+\Delta$. As a result, M also recommends its product for all $\tau > b + c + \Delta$.

Comparing the above cases with and without imitation at stage 3, we conclude that M always prefers to imitate at stage 3 whenever S participates. S's post-participation profit is always $\mu\Delta - K(\Delta)$. Meanwhile, if S does not participate then it sells to direct consumers only and again earns $\mu\Delta - K(\Delta)$. Based on our selection rule, we select the equilibrium in which S breaks the tie in favor of participating in stage 2. This reflects that M is better off as a result of S participating, and therefore could always offer a small transfer to S to ensure it participates. Alternatively, S would strictly prefer to participate if there was some sufficient level of horizontal differentiation. Our tie-breaking rule is consistent with the limit scenario in which the extent of horizontal differentiation becomes arbitrarily small.[19]

In stage 1, we note that in principle any τ generates the same profit for M given that it does not affect S's participation decision and M's subsequent profit. In what follows, and without loss of generality, we select $\tau = b + \Delta^L$ because it coincides with M's optimal fee in the more general case of imperfect imitation (as detailed in Section E of the Online Appendix). Then, the equilibrium in the dual mode is characterized as follows:

Proposition 7 *(Dual mode equilibrium with product imitation and steering) M sets $\tau^{dual} = b + \Delta^L$. S sets $\Delta = \Delta^L$ and participates. In the pricing subgame, $p_o^* = c + \Delta^L$ and $p_m^* = p_i^* = c + \tau^{dual}$. M recommends its own product and all regular consumers buy from M, while $\Pi^{dual} = (1-\mu)\left(b + c + \Delta^L\right)$ and $\pi^{dual} = \mu\Delta^L - K(\Delta^L)$.*

Remarkably, M's equilibrium price $p_m^* = \Delta^L + b + c$ extracts all the innovation surplus, convenience benefit, and cost advantage. There are several important distinctions relative to the dual mode in the baseline model. First, the ability to imitate S's product means M can directly extract the innovation surplus by selling its imitation product rather than doing so indirectly through commissions. For this reason it always engages in self-preferencing in equilibrium. Second, because regular consumers rely on M's recommendations, S's non-participation profit is never higher than the participation profit, so S always participates even though that leads to imitation. Third, M's ability to steer consumers and engage in self-preferencing shields its product away from any competition, allowing it to charge a high price for its imitation product. Finally, the presence of imitation in dual mode reduces S's incentive to innovate. It always chooses the low innovation level Δ^L in dual mode, as opposed to in the marketplace mode, where it has the highest incentive to innovate and chooses the high innovation level Δ^H.

4.2 Policy interventions

There are two distinct approaches to address the negative implications of product imitation and self-preferencing in dual mode. Consider first the structural approach of banning the dual mode. The first thing to note is that after the ban, M always chooses the seller mode given $\Pi^{sell} \geq \Pi^{market}$, where the inequality is strict if $c > 0$. This is because when consumers rely on M to find out about products, in seller mode M can avoid competition with S, which recall was the main reason M could prefer the marketplace mode over the seller mode in the baseline model. Then, comparing the market outcomes in seller mode and dual mode, we obtain:

Proposition 8 *(Ban on dual mode with imitation and steering) A ban on the dual mode results in M choosing the seller mode, with Π and W decreasing; π, Δ, CS, CS_{regular}, and CS_{direct} remaining unchanged.*

[19] We formally examine this issue in Section E of the Online Appendix.

Surprisingly, even though in dual mode we allow M to freely imitate S's superior product and to steer consumers, and we take into account the effect of this through S's choice on how much to innovate, a ban on dual mode lowers welfare and does not increase consumer surplus. The fact that M switches to seller mode means (i) S is still selling to direct consumers only so its innovation incentive is not improved, and (ii) M still does not compete with S. Meanwhile, the decrease in welfare reflects the loss in convenience benefits for regular consumers who in dual mode buy the superior product through M.

Instead of banning the dual mode, an alternative approach is to impose behavioural remedies such as (i) banning imitation; (ii) banning steering (i.e. require M to provide objective and unbiased recommendations); or (iii) banning both imitation and steering. We now consider the implications of these remedies.

Ban on imitation only. When product imitation is banned, in the equilibrium of the dual mode M optimally sets a high τ and then recommends S's product (because doing so allows the highest-valued product to be sold on the platform, which M can extract through its commission). Without imitation, whenever S participates it sells to all consumers, hence it has the full incentive to innovate, resulting in profit $\Delta^H + (b-\tau)(1-\mu) - K(\Delta^H)$. If it does not participate, its profit is $\mu\Delta^L - K(\Delta^L)$. Hence, M sets the highest possible fee subject to S's participation constraint, i.e. $\tau^{dual} = b + \bar{\Delta}$, where

$$\bar{\Delta} \equiv \frac{\Delta^H - \mu\Delta^L - \big(K(\Delta^H) - K(\Delta^L)\big)}{1-\mu} \in (\Delta^L, \Delta^H),$$

and $\Pi = (b+\bar{\Delta})\,(1-\mu)$. Comparing this outcome to Proposition 7:

Proposition 9 *(Ban on product imitation). A ban on imitation results in M continuing in the dual mode, with Δ increasing; π, CS_{regular}, CS_{direct}, and CS remaining unchanged; Π and W increasing if $c < \bar{\Delta} - \Delta^L$, not changing if $c = \bar{\Delta} - \Delta^L$, and decreasing if $c > \bar{\Delta} - \Delta^L$.*

The proposition has interesting implications. Banning imitation restores S's innovation incentive. The exact welfare gain from improved innovation is $\Delta^H - \Delta^L - \big(K(\Delta^H) - K(\Delta^L)\big)$, or $\big(\bar{\Delta} - \Delta^L\big)(1-\mu)$. At the same time, banning imitation prevents consumers from being able to access S's superior product via M's imitation of it, which means M's cost advantage is not realized. This results in the welfare loss of $c\,(1-\mu)$. Therefore, when $c < \bar{\Delta} - \Delta^L$, welfare increases following a ban on imitation, and vice-versa. Nonetheless, whenever a ban on imitation leads to a welfare loss, it is easily verified that the loss is always smaller than the corresponding welfare loss from a ban on dual mode.

Following the same logic as above, M can also benefit from a ban on imitation. This happens when $c < \bar{\Delta} - \Delta^L$ because it is profitable for M to induce the high innovation level and partially extract it through its commission. When $c \geq \bar{\Delta} - \Delta^L$, M does not benefit much from inducing S to choose the high level of innovation. Instead, it is better off directly imitating S's (low-innovation) product and making all the sales by itself while leveraging its own cost advantage. Thus, M benefits from a ban on imitation only when it rules out imitating products involving sufficiently large innovations. In practice, M could try to replicate the effect of such a policy itself by building up a reputation for only copying products based on relatively minor innovations.

Banning imitation does not improve consumer surplus because steering is still present. Recall that in the baseline model where regular consumers know about all products, M's commission level was capped by the so-called "showrooming constraint" (Wang and Wright, 2020): when τ is too high, in the pricing subgame S sets prices to attract consumers to its direct channel in order to avoid paying the commission. When M can steer regular consumers, S knows that if it were to set such prices, then M would recommend the product it sells instead of S's product. The threat of not being recommended means S does not attempt to attract consumers to the direct channel even when the commission τ is high. Even though in the overall equilibrium M still recommends S, the possibility of biased product recommendations (off-equilibrium) removes the showrooming constraint on M's commission. Moreover, given M bases its recommendation on the revenue generated rather than what is best for consumers, price competition in the marketplace is weak, leading to a high inside price, namely $p_i^* = b + c + \bar{\Delta}$.

Finally, one could also consider the option of forcing M to share with all third-parties any proprietary data it gains from observing the sales of a particular third-party seller on its marketplace. This would be a way to ensure that M does not have any advantage in imitating S. However, even with such a data-sharing policy, M always wants to imitate S whenever S participates. This is because data sharing does not affect M's profitability. By self-preferencing through the recommendations, M can shield its product from competition with all third-parties. At the same time, data sharing means that now all fringe sellers can perfectly imitate S, which eliminates S's initial profit from the direct channel. As a result, in equilibrium S will not want to participate on the marketplace, so the data sharing policy makes the dual mode non-viable and leads to the same de facto outcome as a ban on the dual mode.

Ban on steering only. When steering is banned, M always recommends the product that provides the highest surplus to consumers. The model unfolds as if regular consumers are fully aware of all offers available in the market whenever S participates, which restores the on-platform competition and cross-channel competition (given that regular consumers are now always aware of the direct channel). In equilibrium M sets $\tau^{dual} = b + \mu\Delta^L$, S sets $\Delta = \Delta^L$ and participates, $p_o^* = c + \Delta^L$ and $p_m^* = p_i^* = c + \tau^{dual}$, and all regular consumers buy from M. Profits are $\Pi = (b + \mu\Delta^L + c)(1 - \mu)$ and $\pi = \mu\Delta^L - K(\Delta^L)$. Comparing this outcome to Proposition 7, we obtain the following result.

Proposition 10 *(Ban on steering). Requiring objective recommendations results in M continuing in the dual mode, with CS_{regular} and CS increasing; Π decreasing; Δ, π, CS_{direct}, and W remaining unchanged.*

Even though M still makes all the inside sales due to its product imitation, banning steering restores competition and so imposes a stronger constraint on M's pricing. The lower price benefits consumers. Nonetheless, banning steering alone does not restore S's innovation incentive, and so total welfare remains unchanged.

Ban on both imitation and steering. If both imitation and steering are banned, then whenever M chooses the dual mode we would recover the dual mode equilibrium from Proposition 3 in the baseline setup, except that we have to take into account endogenous innovation. However, due to the fact M's marketplace enables consumers to discover S's superior product, M is sometimes better off choosing the seller mode rather than the dual mode. Taking both possibilities into account, we obtain the following result.

Proposition 11 *(Ban on product imitation and steering)*

- *If* $\min\left\{\frac{(b+c)\mu}{1-\mu}, \mu\bar{\Delta}\right\} < c$, *banning imitation and requiring objective recommendations results in the same implications as in Proposition 8.*
- *If* $\min\left\{\frac{(b+c)\mu}{1-\mu}, \mu\bar{\Delta}\right\} \geq c$, *banning imitation and requiring objective recommendations results in* M *continuing to choose the dual mode, with* Π *decreasing;* π, Δ, CS, *and* CS_{regular} *increasing;* CS_{direct} *remaining unchanged;* W *increasing if* $c < \bar{\Delta} - \Delta^L$, *not changing if* $c = \bar{\Delta} - \Delta^L$, *and decreasing if* $c > \bar{\Delta} - \Delta^L$.

When imitation and steering are banned, M prefers operating in the seller mode if

$$\min\left\{\frac{(b+c)\,\mu}{1-\mu}, \mu\bar{\Delta}\right\} < c.$$

The condition is more likely to be satisfied if c is large (provided $\mu < 1/2$), or if $\bar{\Delta}$, b, or μ are small. This reflects that in dual mode, M effectively promotes S's superior product to regular consumers who were previously unaware of S's existence. Since M cannot steer consumers and since the product it sells is inferior, S can potentially set a low outside price to induce these consumers to purchase through the outside channel. This showrooming constraint caps M's ability to extract innovation surplus through its commission. Hence, it prefers to directly profit from its own efficiency advantage (operating as a seller) when c is large. If instead

$$\min\left\{\frac{(b+c)\,\mu}{1-\mu}, \mu\Delta\right\} \geq c,$$

then M continues to operate in the dual mode even after imitation and steering are banned. In this case the intervention restores both S's innovation incentive and competition, improving consumer surplus. Meanwhile, the changes in welfare reflect the same trade-off as in Proposition 9.

Comparing Proposition 8 and Propositions 9 - 11, we note that in our model, targeted behavioural remedies lead to better outcomes than the structural remedy of banning the dual mode altogether. Indeed, the post-intervention equilibria with behavioural remedies generally have weakly higher consumer surplus and welfare, relative to the post-intervention equilibrium with an outright ban on the dual mode. We provide some further thoughts on the practicalities of behavioural remedies in the concluding section.

5 Extensions

We consider several different extensions of the baseline model to explore how the effects of banning the dual mode change. We focus on the main results, with detailed derivations and proofs of propositions relegated to the Online Appendix.

5.1 Multiple product categories

In this section, we confirm that our main result (Proposition 6) goes through largely unchanged when we extend the model to allow for $n > 1$ product categories. Consistent with our discussion in the introduction, we still only consider dual mode bans at the individual product level.

Suppose there are $n > 1$ product categories $j \in \{1, ..., n\}$. In each category, there is a superior seller S_j with competitive advantage Δ_j. We assume that, unless the dual mode is banned, M cannot commit not to enter and compete in any individual product category, except in the extreme case when M chooses the marketplace mode for all product categories. Indeed, in this case, M can be viewed as a pure marketplace without any product storage and shipping capabilities (e.g. eBay, Etsy). However, as soon as M competes in at least one product category, it cannot commit not to also compete in any other category. Thus, when the dual mode is allowed, M's options are:

1. commit to the pure marketplace mode for all product categories.
2. commit to the pure seller mode for some or all product categories (since M can commit not to allow third-parties to sell by not providing that option on its website for those products), and act in dual mode for the rest of the categories (whether M chooses to sell itself is determined by the price it sets).

In contrast, if the dual mode is banned, then M can choose the marketplace mode for some product categories and the seller mode for the rest.

To complete the model in a way that is consistent with our benchmark setting, we assume that b is incurred on a per-transaction basis, meaning that if a consumer buys n products on M then the total convenience benefit is nb. Finally, we assume M can set τ_j separately for each product category $j \in \{1, ..., n\}$. These assumptions mean that each category unfolds independently, and so the analysis of the subgame in each mode remains the same as above.

Without any business model restrictions, M prefers to operate in the dual mode for all product categories. When the dual mode is banned, M chooses the marketplace mode for all categories such that $\Delta_j \geq \frac{c}{1-\mu}$, and the seller mode for all categories such that $\Delta_j < \frac{c}{1-\mu}$. To assess the overall effect of the ban, we simply need to sum up its effect over all product categories. Without loss of generality, we order product categories such that $\Delta_1 \leq \Delta_2 \leq ... \leq \Delta_n$. Let $\tilde{r} \equiv \max\left\{j|\Delta_j < \frac{c}{1-\mu}, j \geq 0\right\}$, i.e., $\tilde{r}$ indicates the number of categories in which M acts as a seller after the ban. There are two cases for the post-ban outcome :

- If $\tilde{r} = 0$, M switches to the marketplace mode for all n categories after the ban, with Π, $CS_{regular}$, and CS decreasing; π increasing; and CS_{direct} and W not changing.
- If $\tilde{r} > 0$, M switches to the seller mode for categories $j \leq \tilde{r}$, and switches to the marketplace mode for categories $j > \tilde{r}$. We have Π, $CS_{regular}$, and W decreasing; π and CS_{direct} increasing; Meanwhile, the net change in total CS is

$$(1-\mu)\sum\nolimits_{j\leq\tilde{r}}(1-\eta_j)(\Delta_j - b - c) - (1-\mu)\sum\nolimits_{j>\tilde{r}}\min\{b+c,(1-\mu)\Delta_j\},$$

where η_j denote the probability that regular consumers buy from M in the equilibrium in the seller mode of category j. Therefore, the ban increases CS if $\Delta_j < b + c$ for all $j \leq \tilde{r}$, and decreases CS if $\Delta_j > b + c$ for all $j \leq \tilde{r}$ and $\tilde{r}$ is sufficiently large relative to n.

These results and insights are essentially the same as those obtained in Proposition 6 — the ban on dual mode reduces consumer surplus and/or welfare.

5.2 Competition and endogenous market structure

In the baseline model, there is always a single platform that optimally chooses to operate in dual mode, absent any policy restrictions. In this section, we consider the entry decisions of multiple platforms and show how the market structure assumed in our baseline model can arise endogenously.

Suppose there are two homogenous platforms M_1 and M_2. In stage 0, M_1 first chooses whether to enter the market, which requires incurring some small fixed cost $F > 0$, and if it enters, the mode it wants to operate in. After observing M_1's choice, M_2 also chooses whether to enter, which requires incurring the same small fixed cost F.[20] After the entry stages, both platforms choose their decision variables according to our existing timing assumptions, competing between themselves and with S. Specifically, if both platforms enter the market, the timing is modified as: (1) for every $i \in \{1, 2\}$ such that M_i does not operate as a pure seller, M_i sets commission τ_i; (2) for every $i \in \{1, 2\}$ such that M_i does not operate as a pure seller, S chooses whether to participate on marketplace i; (3) All sellers, S, and every M_i that is not operating as a pure marketplace for $i = 1, 2$ simultaneously set prices. Note S is free to join one or both marketplaces (when both are available), and regular consumers can freely choose between all channels. We could also allow S to face a multihoming cost: this does not affect the main conclusion below.

We start with the following observations. First, if both platforms operate as pure sellers, then in equilibrium both platforms earn zero profit by the logic of symmetric Bertrand competition. Second, if both platforms operate in the marketplace mode or in dual mode, then S optimally sells to regular consumers exclusively through whichever platform sets a lower commission because S can always adjust its relative prices to divert all regular consumers to the higher-margin channel. Anticipating this, both platforms compete their fees down to zero. Third, if exactly one platform operates as a pure seller and the other platform operates in one of the other two modes, then the analysis follows the case of the functional separation mode studied in Section C of the Online Appendix, with the equilibrium profit of the pure seller being zero.

Completing the discussion above, in Section F of the Online Appendix we derive the following table that summarizes both platforms' post-entry profits for all possible combinations of modes, where the first and second entries in each box represent M_1 and M_2's profits, gross of the fixed

[20] Alternatively, we can assume that M_1 and M_2 make simultaneous entry decisions, which does not affect the result we obtain.

entry cost F.

	M_2 marketplace	M_2 seller	M_2 dual
M_1 marketplace	$0, 0$	$\tau^*(1-\mu), 0$	$0, 0$
M_1 seller	$0, \tau^*(1-\mu)$	$0, 0$	$0, \tau^*(1-\mu)$
M_1 dual	$0, 0$	$\tau^*(1-\mu), 0$	$0, 0$

, (4)

where $\tau^* = \min\left\{\Delta - c, \frac{b+c\mu}{1-\mu}\right\} > 0$. Clearly, once M_1 has entered the market in dual mode (the most profitable mode), the fixed cost $F > 0$ means M_2 has no incentive to enter. Inspecting Table 4 leads to the following result:

Proposition 12 *(Multiple platforms)*

- *In the overall equilibrium M_1 enters the market and operates in dual mode, while M_2 does not enter. The equilibrium prices are therefore the same as in Proposition 3.*
- *If the dual mode is banned, M_1 enters in marketplace mode, leading to lower CS_{regular}, CS and Π; higher π; and unchanged CS_{direct} and W.*

If the dual mode is banned, then M_1 optimally enters as a pure marketplace. Doing so has two competitive advantages over entering as a pure seller. First, there is a first-mover advantage due to fees being set before prices. Second, in the equilibrium of the post-entry subgame the platform that creates higher total value attracts all regular consumers, and operating in marketplace mode indeed creates a higher value (than the seller mode) by combining S's superior product with M_1's convenience benefit. Thus, when there is potential competition between platforms, banning the dual mode results in M_1 always choosing the marketplace mode.

An important implication of Proposition 12 is that the the existence of a potential competing platform means that M never switches to pure seller mode following a ban on the dual mode. Hence, competition mitigates the potential welfare loss from the ban stated in Proposition 6, but the the loss in consumer surplus due to the switch to the marketplace mode persists.

5.3 Comparison with wholesaler-retailer model

At a high level, Amazon's practice of selling on its own marketplace appears similar to that of retailers (e.g. Costco, Home Depot, Target, Trader Joe's, WalMart) that offer their own in-house brands alongside products sourced from third-party suppliers. In this section, we discuss the key differences between these two practices and why the lessons from one do not translate to the other (see also Johnson, 2020, who discusses additional differences).

A key difference in the wholesaler-retailer structure compared to Amazon's use of dual mode is the degree of competition between intermediaries. Amazon is widely perceived to dominate online retail sales in the U.S. and some other countries, whereas there is strong competition between supermarket chains in most jurisdictions. As highlighted in the previous section, the effect of banning the dual mode can be quite different with and without competing platforms, suggesting one reason why the lessons from retailers' use of house brands do not translate to Amazon's use of dual mode.

Arguably a more fundamental difference between the two structures is that in the wholesaler-retailer structure the intermediary sets retail prices for all products regardless of whether they are in-house brands or sourced from third parties, whereas when Amazon uses the dual mode, third-party sellers on the marketplace maintain control of their prices. To analyze the implications of this difference, we first lay out a model of the wholesaler-retailer structure that captures this feature. Suppose M is a retailer, and it can source products from S and fringe sellers. In addition, M can also source from its own in-house brand, the marginal cost of which is set at zero as in the baseline model. Then, M sells all sourced products through its own sales channel, competing against the direct channels of S and the fringe sellers. Thus, M is now a multi-product firm setting prices for all its products.

M can choose to operate in three possible modes. Corresponding to marketplace mode, seller mode and dual mode in our benchmark setting, we distinguish between the "third-party products" mode (M sources products from third-parties exclusively), the "in-house products" mode (M source products from its in-house brand exclusively) and the "dual products" mode (M sources both types of products). The specifications of consumer types and utilities are the same as in the baseline model. To make things comparable to the baseline model, we adopt the following timing: (0) M chooses the mode of operation; (1) S and fringe sellers set their wholesale prices simultaneously; (2) M, S, and fringe sellers compete in retail prices.[21]

In Section G of the Online Appendix, we derive the equilibrium of this wholesaler-retailer model under each of the three modes, and show that M prefers the dual products mode in the absence of any intervention. Then, banning the dual-products mode in this wholesaler-retailer model has the following implications:

Proposition 13 *(Ban on dual products mode in the wholesaler-retailer model)*

- *If $\Delta \geq \frac{c}{1-\mu}$, a ban on the dual products mode results in M choosing the third-party products mode, with Π, CS_{regular}, and CS decreasing; π and W increasing; and CS_{direct} not changing.*

- *If $\Delta < \frac{c}{1-\mu}$, a ban on the dual products mode results in M choosing the in-house products mode, with Π, π, and W decreasing; CS_{regular}, CS_{direct}, and CS increasing.*

A few remarks are in order. First, the cutoff in Δ that determines M's choice of mode after the ban is the same as in the baseline model (Proposition 6). This reflects that, in both third-party products mode and in-house products modes, the market outcomes (in terms of retail prices and split of profits) are the same in the baseline model and in the wholesaler-retailer context.

Second, in the wholesaler-retailer model, a new result is that whenever the ban on the dual products mode results in M choosing the third-party products mode, total welfare increases.

[21] In Section G.5 of the Online Appendix we consider an alternative setup in which in stage (1) M sets the wholesale price paid by M to third-party suppliers in case it wants to source from them. Given the wholesale price offered by M, suppliers just decide whether to supply M or not. We show that the overall insights are broadly similar in this case.

Indeed, the dual products mode involves a mixed-strategy equilibrium, in which not all regular consumers purchase S's product sold by M. This reflects that M decides all retail prices in the inside channel so there is no competition in the inside channel at the retail level, and that S has an incentive to exploit direct consumers in the outside channel. This parallels the feature of the seller mode in the baseline mode that M and S must randomize over their respective prices in any equilibrium. The fact M's inside price is sometimes high deters regular consumers from buying inside, generating welfare losses.

Third, whenever the ban on the dual products mode results in M choosing the in-house products mode, consumer surplus always increases. This result is driven by the fact that both outside and inside prices are higher in the dual products mode than in the in-house products mode. The outside price is higher because in the dual products mode S partially internalizes the revenue of M's inside sales via its wholesale price, meaning that S would be less aggressive in setting its outside prices. This in turns relaxes the inter-channel competition, allowing M, whose price is not constrained by within-channel competition, to charge a higher inside price than the inside price in the in-house products mode.

6 Discussion and conclusion

The practice of platforms selling products or services alongside offerings from third-party sellers is increasingly widespread. Indeed, such dual mode intermediation has clear benefits across different products: allowing each product to be provided by the more efficient seller (the platform or the third-parties), saving on search costs for consumers, internalizing cross-product spillovers in marketing, etc. And we have shown that there are also benefits of the dual mode even when restricting attention to the same product: combining the higher-quality third-party seller product with the platform's more efficient channel, and exerting some competitive pressure on the third-party sellers. It is therefore not surprising that there are now companies like Mirakl (https://www.mirakl.com/), which help retailers create marketplaces for third-party sellers to sell alongside the products the retailers already sell. Mirakl's customers include Best Buy, Carrefour, Darty, Office Depot, Urban Outfitters and others.

However, the use of dual mode by platforms has also raised concerns from competition authorities regarding the possibility of distorting competition to the disadvantage of third-party sellers, particularly when the platform is a dominant e-commerce firm like Amazon. While such concerns are valid, a blanket ban on the dual mode (i.e. forcing platforms to choose the same mode for all products) is likely to do more harm than good. And even when considering a ban on the dual mode at the individual product level, our analysis suggests that such a ban often benefits third-party sellers at the expense of consumer surplus or total welfare. The main reason for this is that in the dual mode the presence of the platform's products constrains the pricing of the third-party sellers on its marketplace, which benefits consumers. Furthermore, when the third-party products do not offer high net surplus, we have shown that a ban on operating in the dual mode leads the platform to stop operating as a marketplace in the relevant product categories and focus on selling itself, which in turn leads to lower consumer surplus and total welfare.

One may expect that these results would be overturned once we take into account the third-party sellers' incentives to innovate, the possibility for the platform to copy such innovations, and the possibility of self-preferencing in dual mode. Perhaps surprisingly, even in this richer setting, a ban on the dual mode is not necessarily good for consumers or welfare, mainly because such a ban causes the platform to switch to selling itself in order to exploit its role in facilitating product discovery.[22] As such, banning dual mode does not restore the third-party seller's innovation incentive or effective price competition between products, which are the main harms of the practices of product imitation and self-preferencing.

Compared to banning the dual mode outright, we have shown that policy interventions that target specific behaviors by the platform are preferable. Namely, banning the imitation of (highly) innovative third-party products by the platform restores sellers' incentive to innovate while still preserving the various benefits associated with the dual mode. Similarly, a ban on self-preferencing preserves the benefits of the dual mode, while preventing the platform from charging a higher price on the product it sells or extracting excessively high commissions from third-party sellers.

Of course, a downside of these types of behavioral policy remedies (relative to a broad stroke ban on the dual mode) is that they require continued monitoring of the platform's conduct to be effective. For example, banning imitation would be hard to implement in practice. This is despite the fact that, as shown in our paper, the platform has an incentive to commit itself not to imitate highly innovative third-party products in order to preserve their incentives to innovate, and so would potentially benefit from an appropriately implemented ban. The difficulty comes from the fact that in practice, the platform's own employees (working in its in-house products division) may want to opportunistically make use of data from its marketplace division. Interestingly, Amazon has an internal policy forbidding the use of non-public data about specific sellers to launch its own in-house products, and yet, as noted in Mattioli (2020), there are reports of its employees violating the policy. This suggests regulators may require the relevant platforms (e.g. Amazon) to maintain a "Chinese wall" between their respective private label and marketplace divisions, with strict penalties for violations. Similarly, to prevent self-preferencing, the platform may be required to provide public APIs that allow approved outsiders (e.g. policy makers or researchers) to audit their recommendation algorithms.

While in our extensions we tried to capture some other important features of dual mode intermediation (e.g. multiple products or multiple platforms), the analysis was not meant to be exhaustive. One useful direction to further extend the analysis would be to develop a richer model of consumer search. For instance, the dual mode potentially makes it more efficient for consumers to search for their ideal product and seller, both because consumers have access to all sellers and products in one place and because it allows the platform to provide higher quality recommendations of which other related products or sellers to consider. Furthermore, by ensuring there is always sufficient competition for any item, the dual mode allows the platform to commit to leave consumers with sufficient surplus from searching on the platform in the first place. On the other hand, taking into account how consumers search for products and sellers,

[22] Given our motivating examples, we have focused the analysis on the case in which the platform has reached a large enough scale such that consumers rely on it to discover products. For a newly-launched platform, such a discovery role does not exist and the analysis would be closer to the baseline model instead.

the dual mode may introduce additional distortions in the way the platform displays products and sellers on its website, and makes recommendations about which other products and sellers consumers should consider, that go beyond the extreme type of self-preferencing we focused on. Finally, in some practical scenarios the platform may not be aware of the existence or the profitability of certain product categories if third-party sellers are not selling on its marketplace. In such cases, starting off in marketplace mode becomes more profitable all other things equal, because it reduces the hold-up risk for sellers, so that they are more likely to participate, which in turn enables the platform to discover new profitable products (which it may decide to sell itself afterwards). Exploring such an issue requires a dynamic model which goes beyond the scope of the current paper.

7 Appendix

7.1 Proofs for the baseline model

7.1.1 Proof of Proposition 2

In this proof we let $r \in [0,1]$ denote the mass of regular consumers who break ties in favor of M's offering when they are indifferent between M's offering and S's products. We first note the following two lemmas.

Lemma 4 *In seller mode, there is no pure-strategy equilibrium when* $\Delta < \frac{b+c}{1-\mu}$.

Proof. Suppose S sets a deterministic price p_o^*. If $p_o^* = c + \Delta$, then, M must optimally set $p_m^* = c + b$ (if $r = 1$) or $p_m^* = c + b - \epsilon$ for $\epsilon > 0$ small (if $r < 1$) to attract all regular consumers. In both cases, given $\Delta > 0$ and $c + b > 0$, S can profitably deviate by undercutting with $p_o = p_m^* - \epsilon$ to attract all regular consumers. If $p_o^* < c + \Delta$ and S is not attracting any regular consumers, then S can deviate from p_o^* by setting $p_o = c+\Delta$, earning a higher margin selling only to direct consumers. Suppose instead $p_o^* < c + \Delta$ and S is attracting some regular consumers. If $p_o^* > \Delta - b$, M can profitably undercut by setting $p_m = p_o^* + b - \Delta - \epsilon$ (for $\epsilon > 0$ small) to attract all regular consumers; If $p_o^* \leq \Delta - b$, S's equilibrium profit is $p_o^* - c \leq \Delta - b - c$, and it can profitably deviate by setting $p_o = c + \Delta$, exploiting direct consumers to earn $\mu\Delta > \Delta - b - c$. ■

Lemma 5 *In seller mode, there is no equilibrium in which* M *sells to all regular consumers.*

Proof. In any such equilibrium, S necessarily sets $p_o^* = c + \Delta$ given that M, by assumption, is selling to all regular consumers. Then, M must optimally set $p_m^* = c + b$ (if $r = 1$) or $p_m^* = c + b - \epsilon$ for $\epsilon > 0$ small (if $r < 1$). In both cases, given $\Delta > 0$, S can profitably deviate by undercutting with $p_o = p_m^* - \epsilon$ to attract all regular consumers. ■

Consider the equilibrium where S makes all the sales and $p_o^* = \Delta - b$ and $p_m^* = 0$. From the text, the price profile is sustainable as an equilibrium if and only if $\Delta \geq \frac{b+c}{1-\mu}$. When $\Delta < \frac{b+c}{1-\mu}$, Lemma 4 implies there is no other pure strategy with S making all the sales.

We next consider the other type of equilibrium where M sometimes makes a positive amount of sales, noting that such an equilibrium is necessarily in mixed strategies due to Lemma 5. We first suppose $\Delta \leq \frac{b+c}{1-\mu}$ and verify the mixed strategy equilibrium stated in the proposition. The cdf of p_m^*, F_m, is such that S is indifferent for all $p_o^* \in [c + \mu\Delta, c + \Delta]$. Notice when $p_o^* = c + \Delta$, S attracts only direct consumers and obtains profit $\mu\Delta$. Therefore, the indifference condition is

$$(p_o^* - c)\left(\mu + (1-\mu)\left(1 - F_m\left(p_o^* + b - \Delta\right)\right)\right) = \mu\Delta.$$

Letting $p_m^* = p_o^* + b - \Delta$, and rearranging, we get

$$F_m\left(p_m^*\right) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b + \Delta - c}\right),$$

which is exactly the cdf stated in the proposition. Note $F_m\left(c + b - (1-\mu)\Delta\right) = 0$ and

$$\lim_{p_m^* \nearrow c+b} F_m\left(p_m^*\right) = 1,$$

so F_m has no mass point. The cdf of p_o^*, F_o, is such that M is indifferent for all $p_m^* \in [c + b - (1-\mu)\Delta, c+b]$. Notice when $p_m^* = c + b - (1-\mu)\Delta$, M attracts regular consumers with probability one and obtains profit $\left(c + b - (1-\mu)\Delta\right)(1-\mu)$. Therefore, the indifference condition is

$$p_m^*\left(1-\mu\right)\left(1 - F_o\left(p_m^* - b + \Delta\right)\right) = \left(c + b - (1-\mu)\Delta\right)(1-\mu).$$

Letting $p_o^* = p_m^* - b + \Delta$, and rearranging, we get

$$F_o\left(p_o^*\right) = 1 - \left(\frac{c + b - (1-\mu)\Delta}{p_o^* + b - \Delta}\right).$$

Notice $F_o\left(c + \mu\Delta\right) = 0$ and

$$\lim_{p_o^* \nearrow c+\Delta} F_o\left(p_o^*\right) = \frac{(1-\mu)\Delta}{c+b} \leq 1,$$

given $\Delta \leq \frac{b+c}{1-\mu}$. Hence, F_o has a mass point $\Pr\left(p_o^* = c + \Delta\right) = 1 - \frac{(1-\mu)\Delta}{c+b}$ if $\Delta < \frac{b+c}{1-\mu}$. This is exactly the cdf stated in the proposition. Given that F_m has no mass point, any $r \in [0,1]$ can be supported in equilibrium.

Finally, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For S, any $p_o < c + \mu\Delta$ earns profit strictly less than $\mu\Delta$ even if it attracts all consumers and so is dominated by $p_o = c + \Delta$; while any $p_o > c + \Delta$ attracts no consumers due to the existence of fringe sellers. For M, any $p_m < c + b - (1-\mu)\Delta$ is strictly dominated by $p_m = c + b - (1-\mu)\Delta \geq 0$ given that M already attracts regular consumers with probability one at this price; while any $p_m > c + b$ attracts no consumers due to the existence of fringe sellers.

7.1.2 Proof of Lemma 1

Given all regular consumers buy from S through the marketplace, we must have $p_i^* \geq c + \tau$ as otherwise S makes strict losses. Meanwhile, given that $p_m^* \geq 0$ must hold, the best response for S must have $p_i^* \geq \Delta$ since at $p_i^* = \Delta$ it can already attract all regular consumers so would do strictly worse setting a price below this. Therefore, $p_i^* \geq \max\{\Delta, c + \tau\}$. We next establish the upperbound for p_i^*. If S sets its outside price just below $p_o = p_i^* - b$ to attract all regular consumers to the direct channel, its profit would be just below $p_i^* - b - c$, while if S sets $p_o = c + \Delta$ to sell only to direct consumers its profit is $\mu\Delta + \left(p_i^* - c - \tau\right)(1-\mu)$. Comparing these two profit expressions, S does not induce showrooming if and only if the candidate equilibrium inside price of S is

$$p_i^* \leq c + \Delta + \frac{b - (1-\mu)\tau}{\mu} \equiv \hat{p}.$$

Meanwhile, if $p_i^* \leq \tau + \Delta$ then M has no incentive to undercut since all regular consumers are buying through the marketplace in equilibrium and undercutting would require setting $p_m < p_i^* - \Delta \leq \tau$, i.e. a lower margin than what it is earning from fees. Any $p_i^* > \tau + \Delta$ cannot be part of the equilibrium because M would want to undercut the price and get a higher margin selling itself than the commission

τ collected. Therefore, in any marketplace equilibrium, $p_i^* \leq \min\{\hat{p}, \tau + \Delta\}$, and it can be verified that $\min\{\hat{p}, \tau + \Delta\} \leq b + c\mu + \Delta < b + c + \Delta$ so regular consumers indeed do not go outside for $p_i^* \leq \min\{\hat{p}, \tau + \Delta\}$ and $p_o^* = c + \Delta$. Therefore, any

$$p_i^* \in \Phi_i \equiv [\max\{c + \tau, \Delta\}, \min\{\hat{p}, \tau + \Delta\}]$$

with $p_m^* = p_i^* - \Delta$ and $p_o^* = c + \Delta$ can be sustained as an marketplace equilibrium as long as the set Φ_i is non-empty. By construction, any profile with $p_i^* \notin \Phi_i$ cannot be an marketplace equilibrium. Note that $\hat{p} \geq \Delta \Leftrightarrow \tau \leq \frac{b+c\mu}{1-\mu}$ while $\hat{p} \geq c + \tau \Leftrightarrow \tau \leq b + \mu\Delta$. When $\Delta < \frac{b+c}{1-\mu}$, $\tau \leq b + \mu\Delta$ implies $\tau \leq \frac{b+c\mu}{1-\mu}$, so that the set Φ_i is non-empty if and only if $\tau \leq b + \mu\Delta$. When $\Delta \geq \frac{b+c}{1-\mu}$, $\tau \leq \frac{b+c\mu}{1-\mu}$ implies $\tau \leq b + \mu\Delta$, so that the set Φ_i is non-empty if and only if $\tau \leq \frac{b+c\mu}{1-\mu}$.

7.1.3 Proof of Lemma 2

Following the discussion in main text, such equilibria necessarily have $p_o^* = \Delta - b$, $p_m^* = 0$ and $p_i^* \geq \Delta$, and S earns $\pi^* = \Delta - b - c$ in equilibrium. When $\Delta < \frac{b+c}{1-\mu}$, S can profitably deviate by setting $p_o = c + \Delta$ and a high p_i (i.e. deviate to let M win inside), earning deviation profit $\mu\Delta > \Delta - b - c$. When $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau < \frac{b+c\mu}{1-\mu}$, S can profitably deviate by setting $p_o = c + \Delta$ and $p_i = \Delta$ (i.e. deviate by attracting consumers to the marketplace), earning deviation profit

$$\begin{aligned} &= \mu\Delta + (\Delta - c - \tau)(1-\mu) > \mu\Delta + \left(\Delta - c - \frac{b + c\mu}{1-\mu}\right)(1-\mu) \\ &= \Delta - b - c = \pi^*, \end{aligned}$$

where the inequality uses $\tau < \frac{b+c\mu}{1-\mu}$. Finally, when $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau \geq \frac{b+c\mu}{1-\mu}$, S's deviation profit from setting $p_o = c + \Delta$ (while setting either $p_i = \Delta$ or a high p_i that lets M win) is at most

$$\begin{aligned} & \mu\Delta + \max\{\Delta - c - \tau, 0\}(1-\mu) \\ \leq\ & \max\left\{\mu\Delta + \left(\Delta - c - \frac{b + c\mu}{1-\mu}\right)(1-\mu), \mu\Delta\right\} \\ =\ & \max\{\Delta - b - c, \mu\Delta\} \\ \leq\ & \Delta - b - c = \pi^*. \end{aligned}$$

7.1.4 Proof of Lemma 3

We first note the following preliminary lemma:

Lemma 6 *In dual mode, there is no pure-strategy equilibrium in which M makes all the sales to regular consumers.*

Proof. Suppose to the contrary, there is such an equilibrium. Then we must have $p_m^* \leq p_i^* - \Delta$, $p_i^* \leq c + \tau$ (otherwise S will slightly undercut inside), and $p_m^* \leq c + b$ (otherwise regular consumers will buy from fringe sellers outside). Moreover, p_m^* must be such that S prefers to set $p_o^* = c + \Delta$, selling exclusively to the direct consumers, i.e.

$$p_m^* - b + \Delta - c \leq \mu\Delta. \tag{7.5}$$

If $p_i^* - \Delta > c + b$, then M can deviate from (7.5) by raising its price to $p_m = b + c - \epsilon$ (for some small $\epsilon > 0$) while still attracting regular consumers, earning deviation profit

$$\Pi' = (b + c - \epsilon)(1-\mu) > (b + c - (1-\mu)\Delta)(1-\mu) \geq p_m^*(1-\mu) = \Pi^*,$$

where the first inequality uses $\Delta > 0$ while the second inequality uses (7.5). If $p_i^* - \Delta \leq c + b$, then M can deviate from (7.5) by raising its price to $p_m \geq p_i^* - \Delta$, letting S sell inside, and earning deviation profit

$$\Pi' = \tau(1-\mu) \geq (p_i^* - c)(1-\mu) \geq (p_m^* + \Delta - c)(1-\mu) > \Pi^*,$$

where the first two inequalities use $p_i^* \leq c + \tau$ and $p_m^* \leq p_i^* - \Delta$, while the last inequality uses $\Delta - c > 0$. ■

To prove Lemma 3, we now construct the mixed-strategy equilibrium with M making a positive amount of sales. Similar to Proposition 2, if $\Delta > \frac{b+c}{1-\mu}$ then there is no equilibrium with M selling to any regular consumers. Therefore, we focus on $\Delta \leq \frac{b+c}{1-\mu}$ in what follows. There are two possible candidate equilibria.

(1) A candidate seller equilibrium such that S never makes sales inside. Note $p_i^* > c + b + \Delta$ means that regular consumers prefer fringe sellers' product sold directly over S's product sold through the marketplace, so M's price is only constrained by $p_m \leq c + b$. Therefore, the verification of the mixed strategy in this case is exactly the same as the second part of Proposition 2. It remains to check that: (i) M cannot deviate from this mixed strategy equilibrium by letting S win inside, which is true because $p_i^* > c + b + \Delta$ means that if M deliberately lets S win inside then all regular consumers purchase from fringe sellers outside; (ii) S has no incentive to deviate and make sales inside. To check (ii), recall in equilibrium S earns $\mu\Delta$. Given $p_m^* \leq c + b$, S can profitably deviate inside if and only if it can set $p_i \in (c + \tau, c + b + \Delta]$ (so that there is a positive probability it makes a positive profit) together with $p_o = c + \Delta$. The set $(c + \tau, c + b + \Delta]$ is empty if and only if $\tau \geq b + \Delta$. Any deviation $p_i \notin (c + \tau, c + b + \Delta]$ either makes no profit or does not affect purchase decisions. We thus conclude that the (mixed-strategy) equilibrium exists if and only if $\tau \geq b + \Delta$.

(2) A candidate seller equilibrium such that S sometimes makes sales inside. Recall we are focusing on $\tau \in [b + \mu\Delta, b + \Delta)$, and the domains for mixed strategies stated in the lemma are:

$$\begin{aligned} p_m^* &\in [c + b - (1-\mu)\Delta, c + \tau - \Delta]. \\ p_o^* &\in [\mu\Delta + c, c + \tau - b] \cup \{c + \Delta\}, \end{aligned}$$

where $p_i^* = c + \tau$. (Note that all values of p_m^* such that $p_m^* \geq c + \tau - \Delta$ are outcome equivalent because if M sets these prices then S will make all inside sales, in which case M earns only from collecting commission τ. Therefore, it is without loss of generality to rule out $p_m^* > c + \tau - \Delta$.)

We first verify the mixed strategy equilibrium stated in the proposition. The cdf of p_m^* is such that S is indifferent for all $p_o^* \in [\mu\Delta + c, c + \tau - b] \cup \{c + \Delta\}$. Notice when $p_o^* = c + \Delta$, S never attracts regular consumers given $p_m^* \leq c + \tau - \Delta < c + b$, so when $p_o^* = c + \Delta$, the outside channel attracts only direct consumers and S obtains profit $\mu\Delta$. Therefore, the indifference condition is

$$(p_o^* - c)(\mu + (1-\mu)(1 - F_m(p_o^* + b - \Delta))) = \mu\Delta.$$

Letting $p_o^* = p_m^* - b + \Delta$, and rearranging, we get

$$F_m(p_m^*) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b + \Delta - c}\right).$$

Note $F_m(c + b - (1-\mu)\Delta) = 0$ and

$$\lim_{p_m^* \nearrow c+\tau-\Delta} F_m = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{\tau - b}\right) < 1$$

given $\tau < b + \Delta$. Hence, F_m has a mass point $\Pr(p_m^* = c + \tau - \Delta) = \frac{\mu}{1-\mu}\left(\frac{\Delta}{\tau - b} - 1\right)$.

The cdf of p_o^*, F_o, is such that M is indifferent for all $p_m^* \in [c+b-(1-\mu)\Delta, c+b]$. Note that F_o has two mass points: $\rho_h \equiv \Pr(p_o^* = c+\Delta)$ and $\rho_l \equiv \Pr(p_o^* = c+\tau-b)$, and they need to be such that M is indifferent between three price levels:

- $p_m = c+b-(1-\mu)\Delta$, and attracts all regular consumers with probability one. This gives M expected profit $(c+b-(1-\mu)\Delta)(1-\mu)$.
- $p_m = c+\tau-\Delta$, letting S win inside. Here, recall that in the equilibrium specification consumers break ties in favor of S's direct channel, so if $p_m = c+\tau-\Delta$, regular consumers buy from S inside only when $p_o = c+\Delta$. This gives M expected profit $\tau\rho_h(1-\mu)$.
- $p_m = c+\tau-\Delta-\epsilon$ for $\epsilon \searrow 0$, so that M makes sales whenever $p_o^* = c+\Delta$ or $p_o^* = c+\tau-b$, earning expected profit $(c+\tau-\Delta)(\rho_h+\rho_l)(1-\mu)$.

Equating these three profit expressions and eliminating the common factor:

$$\tau\rho_h = (c+\tau-\Delta)(\rho_h+\rho_l) = (c+b-(1-\mu)\Delta).$$

$$\implies \rho_h = \frac{c+b-(1-\mu)\Delta}{\tau} \text{ and } \rho_l = \frac{c+b-(1-\mu)\Delta}{c+\tau-\Delta} - \rho_h.$$

Given $\Delta \le \frac{b+c}{1-\mu}$ and $\tau \ge b+\mu\Delta$, obviously $\rho_h \in [0,1]$ and $\rho_l \le 1$. Moreover, $\tau \ge b+\mu\Delta \implies \tau \ge \Delta - c > 0$, so $\rho_l \ge 0$. It remains to verify the cdf for $p_o^* \in [\mu\Delta+c, c+\tau-b)$. The indifference condition is

$$p_m(1-F_o(p_m-b+\Delta)) = c+b-(1-\mu)\Delta$$

$$\implies F_o(p_o) = 1 - \frac{c+b-(1-\mu)\Delta}{p_o-\Delta+b},$$

Obviously, $F_o(\mu\Delta+c) = 0$, and

$$\lim_{p_o \nearrow c+\tau-b} F_o = 1 - \frac{c+b-(1-\mu)\Delta}{c+\tau-\Delta},$$

which is readily verified to equal to $1-\rho_h-\rho_l$.

Next, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For M, any $p_m < c+b-(1-\mu)\Delta$ is weakly dominated by $p_m = c+b-(1-\mu)\Delta \ge 0$ given that $p_o^* \ge c+\mu\Delta$ and F_o has no mass point at $p_o^* = c+\mu\Delta$; while any $p_m > c+\tau-\Delta$ means S wins the inside competition, which is equivalent to setting $p_m = c+\tau-\Delta$. For S, any $p_o < c+\mu\Delta$ earns profit strictly less than $\mu\Delta$; any $p_o > c+\Delta$ attracts no consumer due to the existence of fringe sellers selling directly; while any $(c+\tau-b, c+\Delta)$ does not attract any regular consumers given $p_m^* \le c+\tau-\Delta$ and is dominated by $p_o = c+\Delta$. Finally, S has no incentive to deviate by changing p_i (or changing both prices together) because any $p_i < p_i^* = c+\tau$ leads to a loss, while any $p_i > p_i^*$ does not change its profit.

Furthermore, notice the mass points $\Pr(p_o^* = c+\tau-b)$ and $\Pr(p_m^* = c+\tau-\Delta)$ mean that there is a strictly positive probability for regular consumers to be indifferent between S's product in both channels. If a positive mass of regular consumers break ties in favor of S's inside product, then S can profitably deviate by shifting mass from $p_o^* = c+\tau-b$ to price just below that, so that all regular consumers buy from S directly whenever $p_m^* = c+\tau-\Delta$ is realized, yielding S a margin of $\tau-b > 0$. In this case, F_o cannot be part of the equilibrium.

Notice if $\tau \to b+\Delta$, the two mass points $\Pr(p_o^* = c+\Delta)$ and $\rho_l \equiv \Pr(p_o^* = c+\tau-b)$ combine, in which $\rho_h+\rho_l = \frac{c+b-(1-\mu)\Delta}{c+b}$, while the mass point $\Pr(p_m^* = c+\tau-\Delta) = \frac{\mu}{1-\mu}\left(\frac{\Delta}{\tau-b}-1\right) \to 0$. Therefore, the equilibrium outcome is continuous with the seller equilibrium obtained when $\tau \ge b+\Delta$. Likewise, when $\tau \to b+\mu\Delta$, then $\Pr(p_m^* = c+\tau-\Delta) \to 0$, i.e. M always lets S win inside, so the equilibrium outcome is continuous with the marketplace equilibrium obtained when $\tau \leq b+\mu\Delta$. Finally, when $\tau < b+\mu\Delta$, in the mixed strategy equilibrium the support for F_m, $[c+b-(1-\mu)\Delta, c+\tau-\Delta]$, become an empty set. In that case, Lemma 6 implies there is no other seller equilibrium.

7.1.5 Proof of Proposition 3

We first derive the equilibrium of the non-participation subgame. The only difference with Proposition 2 is that M's price is bounded by $p_m \leq c+\min\{\tau, b\}$ due to the competition with fringe sellers selling inside and outside.

Lemma 7 *(Dual mode, non-participation)*

- *If* $\Delta > \frac{b+c}{1-\mu}$, *then in the equilibrium* $p_o^* = \Delta - b$ *and* $p_m^* = 0$. *All regular consumers purchase from* S *directly. The equilibrium profits are* $\Pi^{np} = 0$ *and* $\pi^{np} = \Delta - b - c$.
- *If* $\Delta \leq \frac{b+c}{1-\mu}$ *and* $\tau > b-(1-\mu)\Delta$, *then in the mixed-strategy equilibrium,* p_m^* *is distributed according to c.d.f* F_m *with support* $[c+b-(1-\mu)\Delta, c+\min\{\tau,b\}]$, *where*

$$F_m(p_m^*) = \begin{cases} \frac{1}{1-\mu}\left(1-\frac{\mu\Delta}{p_m^*-b+\Delta-c}\right) & \text{for } p_m^* \in [c+b-(1-\mu)\Delta, c+\min\{\tau,b\}) \\ 1 & \text{for } p_m^* \geq c+\min\{\tau,b\} \end{cases}.$$

  p_o^* *is distributed according to c.d.f* F_o *with support* $[c+\mu\Delta, c+\Delta+\min\{0,\tau-b\}] \cup \{c+\Delta\}$, *where*

$$F_o(p_o^*) = \begin{cases} 1-\left(\frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta}\right) & \text{for } p_o^* \in [c+\mu\Delta, c+\Delta+\min\{0,\tau-b\}] \\ 1-\left(\frac{c+b-(1-\mu)\Delta}{c+\tau}\right) & \text{for } p_o^* \in (c+\Delta+\min\{0,\tau-b\}, c+\Delta] \\ 1 & \text{for } p_o^* \geq c+\Delta \end{cases}.$$

  The equilibrium profits are $\Pi^{np} = (c+b-(1-\mu)\Delta)(1-\mu)$ *and* $\pi^{np} = \mu\Delta$.
- *If* $\Delta \leq \frac{b+c}{1-\mu}$ *and* $\tau \leq b-(1-\mu)\Delta$, *then in the equilibrium* $p_o^* = c+\Delta$ *and* $p_m^* = c+\tau$. *All regular consumers purchase from* M. *The equilibrium profits are* $\Pi^{np} = (c+\tau)(1-\mu)$ *and* $\pi^{np} = \mu\Delta$.

Proof. The first part follows from the first part of Proposition 2. For the second part, we verify the mixed strategy equilibrium. If $\tau \geq b$, the derivation is the same as the second part of Proposition 2. We consider $\tau \in (b-(1-\mu)\Delta, b)$ in what follows, in which case

$$\begin{aligned} p_m^* &\in [c+b-(1-\mu)\Delta, c+\tau] \\ p_o^* &\in [c+\mu\Delta, c+\Delta+\tau-b] \cup \{c+\Delta\}. \end{aligned}$$

The cdf of p_m^*, F_m, is such that S is indifferent for all $p_o^* \in [c+\mu\Delta, c+\Delta+\tau-b] \cup \{c+\Delta\}$. Notice when $p_o^* = c+\Delta$, S attracts only direct consumers (given $p_m^* < c+b$) and obtains profit $\mu\Delta$. Therefore, the indifference condition is

$$(p_o^*-c)(\mu+(1-\mu)(1-F_m(p_o^*+b-\Delta))) = \mu\Delta.$$

Letting $p_m^* = p_o^*+b-\Delta$, and rearranging, we get

$$F_m(p_m^*) = \frac{1}{1-\mu}\left(1-\frac{\mu\Delta}{p_m^*-b+\Delta-c}\right),$$

which is exactly the cdf stated in the proposition. Note $F_m(c+b-(1-\mu)\Delta) = 0$ and

$$\lim_{p_m^* \nearrow c+\tau} F_m(p_m^*) = \frac{1}{1-\mu}\left(1-\frac{\mu\Delta}{\tau-b+\Delta}\right) < 1,$$

so F_m has a mass point $\Pr(p_m^* = c + \tau) = \frac{\mu}{1-\mu}\left(\frac{\Delta}{\tau - b + \Delta} - 1\right)$.

The cdf of p_o^*, F_o, is such that M is indifferent for all $p_m^* \in [c + b - (1-\mu)\Delta, c + \tau]$. Notice when $p_m^* = c + b - (1-\mu)\Delta$, M attracts regular consumers with probability one and obtains profit $(c + b - (1-\mu)\Delta)(1-\mu)$. Therefore, the indifference condition is

$$p_m^*(1-\mu)(1 - F_o(p_m^* - b + \Delta)) = (c + b - (1-\mu)\Delta)(1-\mu).$$

Letting $p_o^* = p_m^* - b + \Delta$, and rearranging, we get

$$F_o(p_o^*) = 1 - \left(\frac{c + b - (1-\mu)\Delta}{p_o^* + b - \Delta}\right).$$

Notice $F_o(c + \mu\Delta) = 0$ and

$$\lim_{p_o^* \nearrow c+\Delta+\tau-b} F_o(p_o^*) = 1 - \left(\frac{c + b - (1-\mu)\Delta}{c + \tau}\right) < 1,$$

given $\tau > b - (1-\mu)\Delta$. Hence, F_o has a mass point $\Pr(p_o^* = c + \Delta) = \frac{c+b-(1-\mu)\Delta}{c+\tau}$. This is exactly the cdf stated in the proposition.

Finally, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For S, any $p_o < c + \mu\Delta$ earns profit strictly less than $\mu\Delta$ even if it attracts all consumers so it is strictly dominated by $p_o = c + \Delta$; any $p_o > c + \Delta$ attracts no consumers due to the existence of fringe sellers; while any $p_o \in (c + \Delta + \tau - b, c + \Delta)$ does not attract any regular consumers and so is strictly dominated by $p_o = c + \Delta$. For M, any $p_m < c + b - (1-\mu)\Delta$ is weakly dominated by $p_m = c + b - (1-\mu)\Delta \geq 0$ given that $p_o^* \geq c + \mu\Delta$ and F_o has no mass point at $p_o^* = c + \mu\Delta$; while any $p_m > c + \tau$ attracts no consumers due to the existence of fringe sellers in the marketplace. Combining the cases of $\tau \in (b - (1-\mu)\Delta, b)$ and $\tau \geq b$ gives the characterization in the second case of the lemma.

For the third case, when $p_m^* = c + \tau \leq c + b - (1-\mu)\Delta$, S has no incentive to deviate from $p_o^* = c + \Delta$ because its deviation profit is no higher than $p_m^* - b + \Delta - c \leq \mu\Delta$, i.e. lower than its equilibrium profit. M has no incentive to deviate given the constraint $p_m \leq c + \tau$. ∎

Therefore, regardless of τ and equilibrium selection, in all equilibrium in the stage 3 subgame following S's participation, S's profit is weakly higher than $\pi^{np} = \max\{\mu\Delta, \Delta - b - c\}$, so it always participates. From Table 3, we know M does best by setting the highest τ as long as it still induces the marketplace equilibrium in the stage 3 subgame. Hence, $\tau^{dual} = b + \mu\Delta$ when $\Delta \leq \frac{b+c}{1-\mu}$, and $\tau^{dual} = \frac{b+c\mu}{1-\mu}$ when $\Delta > \frac{b+c}{1-\mu}$.

7.1.6 Proof of Proposition 4

Clearly, $\Pi^{dual} > \Pi^{market}$. When $\Delta > \frac{b+c}{1-\mu}$, $\Pi^{dual} = b + c\mu > 0 = \Pi^{sell}$. When $\Delta \leq \frac{b+c}{1-\mu}$, $\Pi^{dual} = (b + \mu\Delta)(1-\mu) > (b + \mu\Delta + c - \Delta)(1-\mu) = \Pi^{sell}$. Moreover, if $\Delta > \frac{b+c}{1-\mu}$, $\Pi^{sell} = 0 < \Pi^{market}$; while if $\Delta \leq \frac{b+c}{1-\mu}$ then $\Pi^{sell} \leq \Pi^{market}$ if and only if $\Delta \geq \frac{c}{1-\mu}$. Given $b \geq 0$, we can combine both possibilities to conclude that $\Pi^{sell} \leq \Pi^{market}$ if and only if $\Delta \geq \frac{c}{1-\mu}$. Finally, $\pi^{market} = \Delta > \pi^{dual} = \max\{\mu\Delta, \Delta - b - c\} = \pi^{sell}$.

7.1.7 Proof of Proposition 5

Let $0 \leq \eta < 1$ denote the probability that regular consumers buy from M in the equilibrium in seller mode, where $\eta > 0$ if $\Delta < \frac{b+c}{1-\mu}$. The associated welfare is $W^{sell} = v + \eta(1-\mu)b + (1 - \eta(1-\mu))(\Delta - c)$, since regular consumers either purchase from M (generating total welfare $v + b$), or along with direct consumers purchase S's product directly (generating total welfare $v+\Delta-c$). In the marketplace mode and the dual mode, all regular consumers buy S's product through M's marketplace, generating total welfare $W^{market}=W^{dual}=v+(1-\mu)(b+\Delta-c)+\mu(\Delta-c)$. Clearly, $W^{sell}\leq W^{market}=W^{dual}$ since $\Delta>c$, with the inequality strict if in addition $b>0$ or $\eta>0$.

Consider consumer surplus. For direct consumers, $CS^{dual}_{direct}=CS^{market}_{direct}=v-c<CS^{sell}_{direct}$ given $p^*_o=c+\Delta$ in the first two modes and given $p^*_o<c+\Delta$ in the seller mode. For regular consumers, we know $CS^{market}_{regular}=v-c$ always. Meanwhile, if $\Delta>\frac{b+c}{1-\mu}$ then

$$
\begin{aligned}
CS^{sell}_{regular} &= v+\Delta-(\Delta-b)=v+b\\
CS^{dual}_{regular} &= v+\Delta+b-\Delta=v+b.
\end{aligned}
$$

If $\Delta\leq\frac{b+c}{1-\mu}$, in the seller mode we can bound $p^*_o\in[c+\mu\Delta,c+\Delta]$, so

$$
\begin{aligned}
CS^{sell}_{regular} &\in (v-c,v-c+(1-\mu)\Delta)\\
CS^{dual}_{regular} &= v+\Delta+b-(c+b+\mu\Delta)=v-c+(1-\mu)\Delta.
\end{aligned}
$$

Therefore, $CS^{dual}_{regular}\geq CS^{sell}_{regular}>CS^{market}_{regular}$. For total consumer surplus, the results follow from summing up the comparisons for each groups of consumers. The only non-trivial case is the comparison between CS^{dual} and CS^{dual} when $\Delta\leq\frac{b+c}{1-\mu}$, in which case we can calculate

$$
\begin{aligned}
CS^{sell} &= W^{sell}-\Pi^{sell}-\pi^{sell}\\
&= v-c+(1-\mu)^2\Delta+(1-\mu)(1-\eta)(\Delta-b-c),
\end{aligned}
$$

and comparing it to $CS^{dual}=v-c+(1-\mu)^2\Delta$.

7.2 Proofs for Section 4

7.2.1 Proof of Proposition 7

We first verify the stated recommendation rule in the stage 4 pricing game in case product imitation hasn't occurred in stage 3. For $\tau<b+c$, any $p_m\in(\tau,b+c]$ ensures M is recommended (by itself in the recommendation stage), and it does best setting $p^*_m=b+c$, i.e. the highest price that avoids consumers switching to the fringe suppliers' product in the direct channel. Any deviation setting $p_m\leq\tau$ means M earns at most the margin τ. Meanwhile, given $p^*_m=b+c$, S knows its inside product is never recommended and so sets $p^*_o=c+\Delta$ to focus on exploiting direct consumers. For $\tau>b+\Delta$, recall if S wants to make any sales inside, its inside price must be at most $p_i=b+c+\Delta$ due to the existence of fringe suppliers' direct product and so $p_i-c-\tau<0$. This means making sales inside is never profitable for S, so for any p^*_o it sets, it would also set $p^*_i>p^*_o+b$ to ensure it does not make sales inside. Given this, the recommendation rule means M never recommends S. Hence, we have $p^*_m=b+c$ and $p^*_o=c+\Delta$ as in the previous paragraph. For $\tau\in[b+c,b+\Delta]$, M now recommends S's product for all $p^*_m\leq b+c$ as long as $p^*_i>p^*_o+b$, due to the higher margin. This means if S sets $p^*_i=c+\Delta+b$ and $p^*_o=c+\Delta$, its profit is $\mu\Delta+(\Delta+b-\tau)(1-\mu)$; note S cannot deviate by setting prices that attract all regular consumers to the direct channel, because it would not get recommended in that case. The only possible deviation for S is to set $p_i>p_o$ to avoid being recommended, which is strictly profitable if and only if $\tau>b+\Delta$. The remaining derivations follow from the analysis in the main text.

7.2.2 Proof of Proposition 8

We first formally state and prove the equilibrium in marketplace mode.

Lemma 8 *(Marketplace with steering) M sets* $\tau^{market} = b$, *S sets* Δ^H, $p_i^* = c + \Delta^H + b$ *and* $p_o^* = c + \Delta^H$. *All regular consumers purchase from S through the marketplace. The equilibrium profits of M and S are* $\Pi^{market} = b(1-\mu)$ *and* $\pi^{market} = \Delta^H - K(\Delta^H)$ *respectively.*

Proof. Fix τ and Δ, and suppose S has joined the marketplace. Recall that fringe suppliers always set inside and outside prices at $c + \tau$ and c. For $\tau > b$, M's recommendation rule given in the main text means it always recommends S's product over a fringe supplier's product. If S sets prices to keep regular consumers in the marketplace, its highest profit is $\Delta - (b - \tau)(1 - \mu)$ which it obtains by setting $p_o = c + \Delta$ and $p_i = c + \Delta + b$; if S sets prices to attract regular consumers to the direct channel, it earns Δ by setting $p_o = c + \Delta$ and $p_i > c + \Delta + b$. The latter is clearly more profitable, implying $\Pi = 0$ in the equilibrium of the subgame following $\tau > b$. For $\tau \leq b$, S is recommended only when it sets $p_i \leq p_o + b$. Adhering to this constraint, S's highest profit is Δ which it obtains by setting $p_o = c + \Delta$ and $p_i = c + \Delta + \tau$. The deviation profit from setting $p_i > p_o + b$ is at most $\Delta\mu$, and so is unprofitable. For all τ, S's non-participation profit in the equilibrium of the subgame is at most $\Delta\mu$, so it always participates and chooses innovation level Δ^H which maximize its post-participation profit $\Delta - K(\Delta)$. Moving to the commission-setting stage, it is clear that M does best setting $\tau = b$ because any $\tau > b$ means S sets prices to attract regular consumers to the direct channel (even though S still participates). ■

For the seller mode, as stated in the main text, the equilibrium has $p_m^* = b + c$ and $p_o^* = c + \Delta$, with profits $\Pi^{sell} = (b + c)(1 - \mu)$ and $\pi^{sell} = \mu\Delta^L - K(\Delta^L)$. Clearly, $\Pi^{sell} \geq \Pi^{market}$, where the inequality holds whenever $c > 0$, so M chooses the seller mode after the ban. The remaining results follow from a direct comparison with the equilibrium in Proposition 7.

7.2.3 Proof of Proposition 9

We first formally state and verify the equilibrium in dual mode when only steering is allowed.

Lemma 9 *(Dual mode equilibrium with steering) M sets* $\tau^{dual} = b + \bar{\Delta}$. *S sets* $\Delta = \Delta^H$ *and participates. In the pricing subgame,* $p_i^* = b + c + \Delta^H$, $p_o^* = c + \Delta^H$, *and* p_m^* *can be any value. M recommends S to all regular consumers, while* $\Pi^{dual} = (1 - \mu)\tau^{dual}$ *and* $\pi^{dual} = \mu\Delta^L - K(\Delta^L)$.

Proof. If M sets $\tau > b + \bar{\Delta}$, S does not participate so M earns pure seller profit $(b + c)(1 - \mu)$. If M sets $\tau \leq b + \bar{\Delta}$, S participates and sets $\Delta = \Delta^H$. Recall M's recommendation rule in this case has been derived in the main text when we analyze the stage 4 pricing game in case product imitation hasn't occurred in stage 3. Therefore, M's profit is $(b + c)(1 - \mu)$ if $\tau < b + c$ and is $\tau(1 - \mu)$ if $[b + c, b + \Delta^H]$. Hence, M's profit is increasing in τ until the participation constraint binds at $\tau = b + \bar{\Delta} < b + \Delta^H$. ■

Clearly, $\Pi^{dual} > \Pi^{sell}$ given $\bar{\Delta} > \Delta^L$, and so M continues to adopt the dual mode after imitation is banned. The remaining results follow from a direct comparison with the equilibrium in Proposition 7.

7.2.4 Proof of Proposition 10

We first formally state and verify the equilibrium of the pricing subgame in dual mode when only imitation is allowed, conditioned on S participating.

Lemma 10 *In stage 4 of the dual mode with only imitation, for each given* (τ, Δ)*:*

- *If* $\tau > b + \mu\Delta$, *in the mixed-strategy equilibrium,* p_m^* *is distributed according to c.d.f* F_m *with support* $[c + \mu\Delta + b, \bar{p}_m]$, *where* $\bar{p}_m = c + \min\{\tau, \Delta + b\}$ *and*

$$F_m(p_m^*) = \begin{cases} \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b - c}\right) & \text{for} \quad p_m^* \in [c + \mu\Delta + b, \bar{p}_m) \\ 1 & \text{for} \quad p_m^* \geq \bar{p}_m \end{cases},$$

p_o^* is distributed according to c.d.f F_o with support $[c+\mu\Delta, \bar{p}_m - b] \cup \{c+\Delta\}$, where

$$F_o(p_o^*) = \begin{cases} 1 - \left(\frac{c+b+\mu\Delta}{p_o^*+b}\right) & \text{for} \quad p_o^* \in [c+\mu\Delta, \bar{p}_m - b] \\ 1 - \left(\frac{c+b+\mu\Delta}{\bar{p}_m}\right) & \text{for} \quad p_o^* \in (\bar{p}_m - b, c+\Delta) \\ 1 & \text{for} \quad p_o^* \geq c+\Delta \end{cases},$$

while $p_i^* \geq c+\tau$. The equilibrium profits are $\Pi = (c+b+\mu\Delta)(1-\mu)$ and $\pi = \mu\Delta - K(\Delta)$.

- *If $\tau \leq b+\mu\Delta$, in the equilibrium, $p_o^* = c+\Delta$ and $p_m^* = p_i^* = c+\tau$. All regular consumers purchase from M. The equilibrium profits are $\Pi = (c+\tau)(1-\mu)$ and $\pi = \mu\Delta - K(\Delta)$.*

Proof. Consider $\tau \leq b+\mu\Delta$. Given $p_m^* = c+\tau \leq c+b+\mu\Delta$, S has no incentive to deviate to a low outside price to attract all consumers to the direct channel because the profit from doing so is $\tau - b$, which is lower than its equilibrium profit $\mu\Delta$. Likewise, S has no incentive to deviate by changing p_i (or changing both prices together) because any $p_i < p_i^* = c+\tau$ leads to a loss, while any $p_i > p_i^*$ does not change its profit. Meanwhile, M has no incentive to deviate given the constraint $p_m \leq p_i^* = c+\tau$, and $p_m^* = c+\tau \geq 0$ so M is profitable. Finally, any equilibrium with $p_i^* < c+\tau$ and $p_o^* = c+\Delta$ is ruled out because such pairs of prices are weakly dominated by $p_i^* = c+\tau$ and $p_o^* = c+\Delta$.

When $\tau > b+\mu\Delta$, the pure-strategy equilibrium stated above does not exist because S can profitably deviate by attracting all consumers to the outside channel. In this case, there is no pure-strategy equilibrium in the pricing subgame. Suppose to the contrary, there is an equilibrium with S setting a deterministic price p_o^*. If $p_o^* = c+\Delta$, then, M must optimally set $p_m^* = c+b$ (if $r=1$) or $p_m^* = c+b-\epsilon$ for $\epsilon > 0$ small (if $r < 1$) to attract all regular consumers. In both cases, given $\Delta > 0$ and $c+b > 0$, S can profitably deviate by undercutting with $p_o = p_m^* - \epsilon$ to attract all regular consumers. If $p_o^* < c+\Delta$ and S is not attracting any regular consumers, then S can deviate from p_o^* by setting $p_o = c+\Delta$, earning a higher margin selling only to direct consumers. Suppose instead $p_o^* < c+\Delta$ and S is attracting some regular consumers. Then for all $p_o^* \geq c$, M can profitably undercut by setting $p_m = c+b-\epsilon > 0$ to attract all regular consumers. Combining all cases, we conclude there is no pure-strategy equilibrium when $\tau > b+\mu\Delta$.

We now verify the mixed-strategy equilibrium stated in the statement of the lemma. The cdf of p_m^*, F_m, is such that S is indifferent for all $p_o^* \in [c+\mu\Delta, \bar{p}_m - b] \cup \{c+\Delta\}$, where $\bar{p}_m = c+\min\{\tau, \Delta+b\}$. Later, we will verify that p_m^* has no mass point at $p_m^* = c+\Delta+b$, and so when S sets $p_o^* = c+\Delta$ it attracts only direct consumers and obtains profit $\mu\Delta$. Therefore, the indifference condition is

$$(p_o^* - c)(\mu + (1-\mu)(1 - F_m(p_o^*+b))) = \mu\Delta.$$

Letting $p_m^* = p_o^* + b$, and rearranging, we get

$$F_m(p_m^*) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{p_m^* - b - c}\right),$$

which is exactly the cdf stated in the proposition. Note $F_m(c+b+\mu\Delta) = 0$ and

$$\lim_{p_m^* \nearrow \bar{p}_m} F_m(p_m^*) = \frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{\min\{\tau, \Delta+b\} - b}\right). \tag{7.6}$$

If $\tau \geq \Delta+b$ then (7.6) equals one, so F_m has no mass point at $p_m^* = c+\Delta+b$; otherwise if $\tau \in (b+\mu\Delta, b+\Delta)$, then (7.6) equals $\frac{1}{1-\mu}\left(1 - \frac{\mu\Delta}{\tau-b}\right) \in (0,1)$, in which case F_m has a mass point $\Pr(p_m^* = c+\tau)$ at $p_m^* = c+\tau < c+\Delta+b$.

The cdf of p_o^*, F_o, is such that M is indifferent for all $p_m^* \in [c+\mu\Delta+b, \bar{p}_m]$. Notice when $p_m^* = c+\mu\Delta+b$, M attracts regular consumers with probability one and obtains profit $(c+b+\mu\Delta)(1-\mu)$.

Therefore, the indifference condition is

$$p_m^* (1-\mu)(1-F_o(p_m^* - b)) = (c + b + \mu\Delta)(1-\mu).$$

Letting $p_o^* = p_m^* - b$, and rearranging, we get

$$F_o(p_o^*) = 1 - \left(\frac{c+b+\mu\Delta}{p_o^* + b}\right).$$

Notice $F_o(c + \mu\Delta) = 0$ and

$$\lim_{p_o^* \nearrow \bar{p}_m - b} F_o(p_o^*) = 1 - \frac{c+b+\mu\Delta}{c + \min\{\tau, \Delta + b\}} \in (0,1).$$

Hence, F_o has a mass point $\Pr(p_o^* = \bar{p}_m - b) = \frac{c+b+\mu\Delta}{c+\min\{\tau,\Delta+b\}}$. This is exactly the cdf stated in the proposition.

Finally, we check that no player can profitably deviate from outside of its support in the stated mixed strategy equilibrium. For S, any $p_o < c + \mu\Delta$ earns profit strictly less than $\mu\Delta$ even if it attracts all consumers, hence it is strictly dominated by $p_o = c + \Delta$; any $p_o > c + \Delta$ attracts no consumer due to the existence of fringe sellers; while any $p_o \in (c + \tau - b, c + \Delta)$ does not attract any regular consumers. S has no incentive to deviate by changing p_i (or changing both prices together) because any $p_i < p_i^* = c + \tau$ leads to a loss, while any $p_i > p_i^*$ does not change its profit. For M, any $p_m < c + b + \mu\Delta$ is weakly dominated by $p_m = c + b + \mu\Delta$ given that M already attracts regular consumers with probability one at this price; while any $p_m > \bar{p}_m$ attracts no consumer due to the existence of S in the marketplace and fringe sellers in the outside channel. Finally, note this mixed-strategy equilibrium exists as long as $\tau \geq b + \mu\Delta$. If $\tau < b + \mu\Delta$, the support for p_m^*, that is, $[c + \mu\Delta + b, c + \tau]$ becomes an empty set, i.e. there is no p_m that can make S indifferent between setting $c + \Delta$ and prices that are lower. ■

Consider S's decision in stage 3. Given that regular consumers are unaware of S if it does not participate, S's non-participation profit is $\Delta\mu - K(\Delta)$, which is the same as its participation profit. Based on our existing selection rule, we select the equilibrium in which S breaks the tie in favor of participating and then chooses Δ^L which maximizes $\Delta\mu - K(\Delta)$ in stage 2. Then, from Lemma 10, M sets $\tau^{dual} = b + \mu\Delta^L$ and so we obtain the overall equilibrium of the dual mode when only imitation is allowed, as stated in the text. Clearly, $\Pi^{dual} = (b + \mu\Delta^L + c)(1-\mu) > \Pi^{sell}$, and so M continues to operate in dual mode after steering is banned. The remaining results follow from a direct comparison with the equilibrium in Proposition 7.

7.2.5 Proof of Proposition 11

We first formally state and verify the equilibrium in dual mode when both imitation and steering are banned (which is analogous to the dual mode in the baseline model, except that now Δ is endogenously determined by S).

Lemma 11 *(Dual mode equilibrium without imitation and steering) M sets $\tau^{dual} = b + \mu \min\left\{\frac{b+c}{1-\mu}, \bar{\Delta}\right\}$, S sets $\Delta = \Delta^H$ and participates. The equilibrium prices are $p_o^* = c + \Delta^H$, $p_i^* = \max\{c + \tau^{dual}, \Delta^H\}$, and $p_m^* = \max\left\{c + \tau^{dual} - \Delta^H, 0\right\}$. All regular consumers buy from S on M. The equilibrium profits are $\Pi^{dual} = \tau^{dual}(1-\mu)$ and $\pi^{dual} = \max\left\{\Delta^H - b - c - K(\Delta^H), \mu\Delta^L - K(\Delta^L)\right\}$.*

Proof. For each given (τ, Δ), the equilibrium of the pricing subgame in stage 4 has been derived in Section 3.3. From Table 3, we know M does best by setting the highest τ as long as it still induces participation by S in stage 2 and the marketplace equilibrium in the stage 3 subgame, so $\tau \leq b +$

$\mu \min\left\{\frac{b+c}{1-\mu}, \Delta^H\right\}$. For each τ in this range, consider S's decision in stage 2. Its non-participation and participation profits are $\mu\Delta^L - K(\Delta^L)$ and $\Delta^H - K(\Delta^H) + (1-\mu)\min\left\{\frac{b-\tau}{\mu}, -c\right\}$ respectively. Suppose $b + \mu c > \tau$, then S participates if and only if $\bar{\Delta} \geq c$, which holds given the definition of $\bar{\Delta} > \Delta^L > c$. If $\tau > b + \mu c$, then S participates if and only if $\tau \leq b + \mu\bar{\Delta}$. Therefore, if $\tau = b + \mu\bar{\Delta}$ then S is exactly indifferent between participating and non-participating, with profit $\mu\Delta^L - K(\Delta^L)$. Finally, if $b + \mu\bar{\Delta} \leq b + \mu\left(\frac{b+c}{1-\mu}\right)$, then $\tau^{dual} = b + \mu\bar{\Delta}$ does not violate the existence of an marketplace equilibrium, and we are done. Otherwise, the participation constraint does not bind and τ^{dual} is pinned down by the the condition for the existence of marketplace equilibrium, i.e. $\tau^{dual} = b + \mu\left(\frac{b+c}{1-\mu}\right)$. ■

Clearly, $\Pi^{dual} \geq \Pi^{sell}$ if and only if $\min\left\{\frac{(b+c)\mu}{1-\mu}, \mu\bar{\Delta}\right\} \geq c$. If M switches to the seller mode after the ban on imitation and steering, then the result of Proposition 11 is obvious. If M continues to operate in the dual mode after the ban on imitation and steering, the changes in surplus and welfare are easily obtained from a direct comparison. The only non-obvious result is the comparison of M's profit. Specifically, the change of M's profit following the ban is

$$\mu \min\left\{\frac{b+c}{1-\mu}, \Delta^H\right\} - \Delta^L - c,$$

which is negative if $\mu\Delta^H \leq \Delta^L$. Convexity implies $K'(\mu\Delta^H) \leq \mu K'(\Delta^H) = \mu$, which implies $\mu\Delta^H \leq \Delta^L$, as required.

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