Online Appendix: Marketplace leakage

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A Two-part tariff

Suppose M can also extract a share $\beta \in [0,1]$ of S's net profits through a fixed (upfront) fee (β can be thought of as M's bargaining power) that is set at the same time as f. We find that provided $\beta < 1$, our insights go through: there is equilibrium leakage and M's profits are increasing in μ.

Since the fixed fee is paid upfront, it doesn't change S's pricing, which is given by

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad f \geq 2\mu \\ v - \frac{f}{2} & \text{if} \quad f \leq 2\mu \end{cases}$$

as before. S's corresponding profit from participating (before fixed fee) is

$$\pi(f) = (p_d(f) - c)\left(\frac{v - p_d(f)}{\mu}\right) + (v - c - f)\left(1 - \frac{v - p_d(f)}{\mu}\right)$$

The corresponding profit for M is

$$\Pi(f) = \begin{cases} \beta\pi(f) & \text{if} \quad f \geq 2\mu \\ f\left(1 - \frac{f}{2\mu}\right) + \beta\pi(f) & \text{if} \quad f \leq 2\mu \end{cases}.$$

Since $f \leq v - c$, we have $\pi(f) > 0$, so S always participates, since its net profit from an ex-ante perspective is $(1-\beta)\pi(f)$.

Thus,

$$\Pi(f) = \begin{cases} \beta(v - \mu - c) & \text{if} \quad f \geq 2\mu \\ f\left(1 - \frac{f}{2\mu}\right) + \beta\left(\frac{f^2}{4\mu} + (v - c - f)\right) & \text{if} \quad f \leq 2\mu \end{cases}.$$

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And we therefore have

$$
\begin{aligned}
f^* &= \min\left\{\frac{2\mu(1-\beta)}{2-\beta}, v-c\right\} \\
\Pi^* &= f^*\left(1-\frac{f^*}{2\mu}\right)+\beta\left(\frac{f^{*2}}{4\mu}+(v-c-f^*)\right) \\
&= \begin{cases} \frac{\mu(1-\beta)^2}{(2-\beta)}+\beta(v-c) & \text{if } \frac{2\mu(1-\beta)}{2-\beta} \leq v-c \\ (v-c)-\frac{(v-c)^2}{2\mu}\left(1-\frac{\beta}{2}\right) & \text{if } \frac{2\mu(1-\beta)}{2-\beta} \geq v-c \end{cases}.
\end{aligned}
$$

Of course, when $\beta = 0$, we obtain the f^* and Π^* from our baseline case in the main text. Meanwhile, when $\beta \to 1$, we obtain

$$
\begin{aligned}
f^* &= 0 \\
\Pi^* &= v-c.
\end{aligned}
$$

This means that if M can extract S's entire profit via a fixed fee, then there is no reason to charge a transaction fee (since it induces leakage), and M obtains the maximum profit $v-c$ (all transactions are conducted on M). However, provided $\beta < 1$, there will be positive leakage in equilibrium. Furthermore, it is easily seen that Π^* is increasing in μ for all $\beta < 1$.

B Ad-valorem fee

Let $0 < \rho < 1$ be the ad-valorem (proportional) fee charged by M, so that S retains $(1-\rho)p_m$ and pays ρp_m to M. Given prices, the buyers' choices are the same. First, note that M will never set ρ so that $(1-\rho)v < c$, i.e. $\rho > 1-\frac{c}{v}$. If it did, S would make a loss when selling through M at the highest possible price of v, and as a result would simply set some price $p_m > v$, so that it makes no sales on M. This in turn implies M would make zero profits in this case. Second, given that $\rho \leq 1-\frac{c}{v}$, S will set $p_m = v$. The logic is the same as before. And third, S sets $p_d \leq v$, again for the same reason as before.

Given these observations, S's pricing problem reduces to setting $p_d \leq v$ to maximize its profit

$$\pi = (p_d-c)\,G(v-p_d)+((1-\rho)v-c)(1-G(v-p_d)).$$

S's optimal p_d is such that $v-\overline{s} \leq p_d \leq v$.

Denote by $p_d(f)$ the unique solution in p_d to the first-order condition (FOC)

$$G(v-p_d)-g(v-p_d)(p_d-(1-\rho)v)=0,$$

so that

$$p_d(\rho) = (1-\rho)v + \frac{G(v - p_d(\rho))}{g(v - p_d(\rho))}.$$

S's profit maximizing price $p_d^*(\rho)$ is then given by

$$p_d^*(\rho) = \begin{cases} v - \overline{s} & \text{if} \quad p_d(\rho) \leq v - \overline{s} \\ p_d(\rho) & \text{if} \quad v - \overline{s} \leq p_d(\rho) \leq v \\ v & \text{if} \quad p_d(\rho) \geq v \end{cases}.$$

The corresponding profit for M is

$$\Pi^* = \max_{\rho \leq 1 - \frac{c}{v}} \left\{ \rho v \left(1 - G\left(v - p_d^*(\rho)\right)\right) \right\}.$$

Assuming $G(s) = \frac{s}{\mu}$ on $s \in [0, \mu]$, and assuming $\rho \leq 1 - \frac{c}{v}$, we have

$$p_d(\rho) = v\left(1 - \frac{\rho}{2}\right)$$

and therefore S's profit maximizing price $p_d^*(f)$ is given by

$$v\rho = 2\mu$$

$$p_d^*(f) = \begin{cases} v - \mu & \text{if} \quad \rho \geq \frac{2\mu}{v} \\ v\left(1 - \frac{\rho}{2}\right) & \text{if} \quad \rho \leq \frac{2\mu}{v} \end{cases}.$$

The corresponding profit for M is

$$\Pi(\rho) = \begin{cases} 0 & \text{if} \quad f \geq \frac{2\mu}{v} \\ \rho v\left(1 - \frac{v\rho}{2\mu}\right) & \text{if} \quad f \leq \frac{2\mu}{v} \end{cases}.$$

Recalling that M will always set $\rho \leq 1 - \frac{c}{v}$ and following the same steps as the proof of Proposition 1, the optimal ad-valorem fee is $\rho^*(\mu) = \min\left\{\frac{\mu}{v}, 1 - \frac{c}{v}\right\}$. Substituting this back into the above pricing and profit formula, we obtain the identical equilibrium pricing $p_d^*(\mu)$ and profit functions $\Pi^*(\mu)$ as in Proposition 1, proving the results are identical.

C Power function distribution

We repeat our baseline analysis when $G(s) = \frac{1}{\mu}s^\alpha$ for $s \in \left[0, \mu^{\frac{1}{\alpha}}\right]$ to allow for any $\alpha > 0$. Note an increase in the parameter μ still corresponds to an increase in switching costs. By increasing μ, we will increase the expected switching cost which is $\frac{\alpha}{1+\alpha}\mu^{\frac{1}{\alpha}}$, with

$G_2(s)$ stochastically dominating $G_1(s)$ if $\mu_2 > \mu_1 > 0$. We obtain the following proposition which generalizes Proposition 1 in the main text to allow for $\alpha \neq 1$.

Proposition 8. *The optimal transaction fee, direct price and marketplace profits are as follows:*

$$\begin{aligned} f^*(\mu) &= \min\left\{\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}}, v-c\right\} \\ p_d^*(\mu) &= v - \min\left\{\left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}, \frac{\alpha}{1+\alpha}(v-c)\right\} \\ \Pi^*(\mu) &= \begin{cases} \left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}} & \text{if } \mu \leq \frac{\alpha^\alpha (v-c)^\alpha}{(1+\alpha)^{\alpha-1}} \\ (v-c)\left(1-\frac{\alpha^\alpha (v-c)^\alpha}{\mu(1+\alpha)^\alpha}\right) & \text{if } \mu > \frac{\alpha^\alpha (v-c)^\alpha}{(1+\alpha)^{\alpha-1}} \end{cases} \end{aligned}$$

The extent of leakage is $\frac{1}{\mu}(v - p_d^*(\mu))^\alpha$. In response to an increase in switching costs (an increase in μ), the marketplace's fee weakly increases, S's direct price weakly decreases, the extent of leakage weakly decreases, and the marketplace's profit increases. There is always positive but partial leakage.

Proof. Assuming $f \leq v - c$, we have

$$p_d(f) = v - \frac{\alpha}{1+\alpha} f$$

and therefore S's profit maximizing price $p_d^*(f)$ is given by

$$p_d^*(f) = \begin{cases} v - \mu^{\frac{1}{\alpha}} & \text{if } f \geq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \\ v - \frac{\alpha}{1+\alpha} f & \text{if } f \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \end{cases}.$$

The corresponding profit for M is

$$\Pi(f) = \begin{cases} 0 & \text{if } f \geq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \\ f\left(1 - \frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha} f^\alpha\right) & \text{if } f \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}} \end{cases}.$$

Noting second-order conditions hold for any $f > 0$, $\alpha > 0$ and $\mu > 0$, we have

$$\arg\max_f \left\{ f\left(1 - \frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha} f^\alpha\right)\right\} = \frac{1+\alpha}{\alpha}\frac{1}{(1+\alpha)^{\frac{1}{\alpha}}}\mu^{\frac{1}{\alpha}} < \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}}.$$

Taking into account that $f \leq v - c$, this implies the level of f^* and Π^* given in Proposition 8.

First, note that Π^* is always increasing in μ. To determine the direct price, note that if $\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq v-c$, then

$$f^* = \frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}},$$

so in this case

$$p_d^* = v - \left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}.$$

If $\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \geq v-c$, then

$$f^* = v - c \leq \frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}} \leq \frac{1+\alpha}{\alpha}\mu^{\frac{1}{\alpha}},$$

so in this case

$$p_d^* = v - \frac{\alpha}{1+\alpha}(v-c).$$

Combining these two results implies p_d^* in Proposition 8. Since $\left(\frac{\mu}{1+\alpha}\right)^{\frac{1}{\alpha}}$ is everywhere increasing in μ, p_d^* is strictly decreasing in μ below a threshold level of μ, but above that threshold, p_d^* is constant in μ.

The equilibrium extent of leakage is given by

$$\frac{\alpha^\alpha}{\mu(1+\alpha)^\alpha}(f^*)^\alpha.$$

Given $f^* = \min\left\{\frac{(1+\alpha)^{1-\frac{1}{\alpha}}}{\alpha}\mu^{\frac{1}{\alpha}}, v-c\right\}$, the extent of leakage is

$$\min\left\{\frac{1}{1+\alpha}, \frac{\alpha^\alpha (v-c)^\alpha}{\mu(1+\alpha)^\alpha}\right\}.$$

This is weakly decreasing in μ; initially constant, and then decreasing in μ. □

D Alternative tie-breaking assumption

Here we redo the analysis of steering under the assumption that if neither seller is offering non-negative surplus to buyers that purchase via M (i.e. $p_m^l > u$ and $p_m^h > v$), then M does not show either seller.

We first prove the following lemma.

Lemma 4 *If* $u - c \leq \frac{2(v-c)}{3}$, *then*

$$f^* = \begin{cases} u - c & \text{if} \quad \mu \leq 2(u-c) \\ \mu & \text{if} \quad 2(u-c) \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}$$

$$\Pi^* = \begin{cases} u - c & \text{if} \quad \mu \leq 2(u-c) \\ \frac{\mu}{2} & \text{if} \quad 2(u-c) \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}$$

If $\frac{2(v-c)}{3} \leq u - c \leq v - c$, *then*

$$f^* = \begin{cases} u - c & \text{if} \quad \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \\ 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \end{cases}$$

$$\Pi^* = \begin{cases} u - c & \text{if} \quad \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)} \end{cases}$$

Proof of Lemma 4

Here too, given f and seller prices, M recommends the seller that induces the least amount of leakage (i.e. with the lowest non-negative difference between price on M and direct price), subject to offering non-negative utility to buyers that buy via M. We define $\overline{p}_d^l(f)$ and $\overline{p}_d^h(f)$ as in the proof of Lemma 3:

$$\overline{p}_d^l(f) = \max_{\substack{p_d \leq u \\ (p_d - c)\frac{\min\{u-p_d,\mu\}}{\mu} + (u-c-f)\left(1 - \frac{\min\{u-p_d,\mu\}}{\mu}\right) \geq 0}} \{p_d\}$$

$$\overline{p}_d^h(f) = \max_{\substack{p_d \leq v \\ (p_d - c)\frac{\min\{v-p_d,\mu\}}{\mu} + (v-c-f)\left(1 - \frac{\min\{v-p_d,\mu\}}{\mu}\right) \geq 0}} \{p_d\}.$$

First, because $\overline{p}_d^l(f)$ and $\overline{p}_d^h(f)$ are defined in the same way, we still have

$$\overline{p}_d^l(f) = \begin{cases} u & \text{if} \quad f \leq u - c \\ u - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2} & \text{if} \quad \begin{matrix} \mu \leq u - c \leq f \text{ or} \\ \mu > u - c \text{ and } u - c \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \end{matrix} \\ -\infty & \text{if} \quad \mu > u - c \text{ and } f > 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right) \end{cases}$$

$$\overline{p}_d^h(f) = \begin{cases} v & \text{if} \quad f \leq v - c \\ v - \frac{f - \sqrt{f^2 - 4\mu(f-(v-c))}}{2} & \text{if} \quad \begin{array}{c} \mu \leq v - c \leq f \text{ or} \\ \mu > v - c \text{ and } v - c \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) \end{array} \\ -\infty & \text{if} \quad \mu > v - c \text{ and } f > 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) \end{cases}$$

Second, S_h still makes all sales in equilibrium by the same reasoning as in the proof of Lemma 3. The only slight difference is when $v - \overline{p}_d^h(f) = u - \overline{p}_d^l(f) = +\infty$, i.e. neither seller can make non-negative profits with positive sales via M. In this case the two sellers set $p_m^l > u$ and $p_m^h > v$, which means M doesn't show either of them and makes zero profits. This means M would never set such an f in equilibrium in the first place.

There are therefore two cases:

Case 1) If $\mu > u-c$ and $f > 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f) = -\infty$, which means S_l has no chance of making non-negative profits. This implies it might as well price at $p_d^l = p_m^l > u$, which makes it irrelevant. In this case, if $\overline{p}_d^h(f)$ is well-defined (i.e. not equal to $-\infty$), then S_h does best by setting $p_m^h = v$ and $p_d^h = p_d^*(f) = v - \min\left\{\frac{f}{2}, \mu\right\}$, so it gets recommended by M, makes positive profits, and M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$. If $\overline{p}_d^h(f) = -\infty$, then S_h cannot make non-negative profits selling through M, so it sets $p_m^h > v$ and M makes zero profits.

Case 2) If $\mu \leq u-c$ or $\mu > u-c$ and $0 \leq f \leq 2\mu\left(1 - \sqrt{1 - \frac{u-c}{\mu}}\right)$, then $\overline{p}_d^l(f)$ exists. In this case, S_l sets $p_m^l = u$ and $p_d^l = \overline{p}_d^l(f)$, while S_h sets $p_m^h = v$ and p_d^h to maximize profits subject to $v - p_d^h \leq u - \overline{p}_d^l(f)$ (so that it is recommended by M), i.e.

$$\begin{aligned} p_d^h &= \arg \max_{p_d \geq v - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2}} \left\{ (p_d - c)\frac{\min\{v - p_d, \mu\}}{\mu} + (v - c - f)\left(1 - \frac{\min\{v - p_d, \mu\}}{\mu}\right) \right\} \\ &= v - \frac{f - \sqrt{f^2 - 4\mu(f - (u-c))}}{2}, \end{aligned}$$

where the last equality follows because $v - \frac{f - \sqrt{f^2 - 4\mu(f-(u-c))}}{2} \geq \max\left\{v - \frac{f}{2}, v - \mu\right\}$ under the conditions that define case 2). Also, we know that at these prices, S_h must make non-negative profits because if $\overline{p}_d^l(f)$ is well-defined, then so is $\overline{p}_d^h(f)$.

This implies M's profit in this case is

$$f\left(1 - \frac{f - \sqrt{f^2 - 4\mu(f - (u-c))}}{2\mu}\right).$$

If M sets $f \leq u - c$, then $\overline{p}_d^l(f) = u$ and $\overline{p}_d^h(f) = v$. In this case, S_l's best chance to be recommended and make non-negative profits is to set $p_d^l = p_m^l = u$. The best response of

S_h is then to set $p_d^h = p_m^h = v$, which ensures that it is recommended by M (we assume M breaks ties in favor of S_h). This leads all buyers to purchase from S_h on M, so M's profits are equal to f. As a result, M does best in this range to set $f = u - c$, yielding a profit equal to $u - c$.

We can therefore restrict attention to $f \geq u - c$.

Suppose $\mu \leq u - c$, so we are in case 2) above. The derivative of M's profit with respect to f is

$$\begin{aligned} \frac{d\left(f\left(1-\frac{f-\sqrt{f^2-4\mu(f-(u-c))}}{2\mu}\right)\right)}{df} &= \frac{-\left(2\mu - f + \sqrt{f^2-4\mu\left(f-(u-c)\right)}\right)\left(f-\sqrt{f^2-4\mu\left(f-(u-c)\right)}\right)}{2\mu\sqrt{f^2-4\mu\left(f-(u-c)\right)}} \\ &\leq 0, \end{aligned}$$

where the last inequality follows because $f > \sqrt{f^2-4\mu\left(f-(u-c)\right)}$ and $u - c \geq \mu$ imply $2\mu - f + \sqrt{f^2-4\mu\left(f-(u-c)\right)} \geq 0$. This means M wants to set f as low as possible subject to $f \geq u - c$. Thus, we have proven that when $\mu \leq u - c$, the optimal solution for M is to set $f^* = u - c$, resulting in $p_d^h = p_m^h = v$, no leakage and $\Pi^* = u - c$.

Now suppose $\mu > u - c$.

- If $u - c \leq f \leq 2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)$, then we are once again in case 2) above. And once again $2\mu - f + \sqrt{f^2-4\mu\left(f-(u-c)\right)} \geq 0$ because $2\mu - f \geq 0$, so M's best option on this range is to set $f = u - c$, resulting in profit $u - c$.
- If $\mu \leq v - c$, then M can set f such that $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) < f \leq 2\mu$ to obtain profits $f\left(1-\frac{f}{2\mu}\right)$ (case 1) above)
- If $\mu > v - c$, then M can set f such that $2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) < f \leq 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$ to obtain profits $f\left(1-\frac{f}{2\mu}\right)$ (case 1) above)

Thus, if $u - c < \mu \leq v - c$, then M chooses between $u - c$ and

$$\max_{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f\leq 2\mu}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\sqrt{1-\frac{u-c}{\mu}}\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) & \text{if } \mu \leq \frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if } \mu \geq \frac{4(u-c)}{3} \end{cases}$$

And if $\mu > v - c$, then M chooses between $u - c$ and

$$\max_{\substack{2\mu\left(1-\sqrt{1-\frac{u-c}{\mu}}\right)<f \\ f\leq 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)}}\left\{f\left(1-\frac{f}{2\mu}\right)\right\} = \begin{cases} 2\mu\sqrt{1-\frac{u-c}{\mu}}\left(1-\sqrt{1-\frac{u-c}{\mu}}\right) & \text{if } \mu \leq \frac{4(u-c)}{3} \\ \frac{\mu}{2} & \text{if } \frac{4(u-c)}{3} \leq \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) & \text{if } \mu \geq \frac{4(v-c)}{3} \end{cases}$$

Suppose $u-c \leq \frac{v-c}{2}$. Then:

- if $u-c < \mu \leq 2(u-c)$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $2(u-c) \leq \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = \mu$, yielding $\Pi^* = \frac{\mu}{2}$.
- if $\mu \geq \frac{4(v-c)}{3}$, the optimal solution is $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$, yielding

$$\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right).$$

Suppose $\frac{v-c}{2} \leq u-c \leq \frac{2(v-c)}{3} < \frac{3(v-c)}{4}$. Then:

- if $u-c < \mu \leq v-c$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $v-c < \mu \leq 2(u-c)$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $2(u-c) < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = \mu$, yielding $\Pi^* = \frac{\mu}{2}$.
- if $\mu \geq \frac{4(v-c)}{3}$, the optimal solution is $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$, yielding

$$\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right).$$

Suppose $\frac{2(v-c)}{3} < u-c \leq \frac{3(v-c)}{4}$. Then $2(u-c) > \frac{4(v-c)}{3}$ and:

- if $u-c < \mu \leq v-c$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $v-c < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = u-c$, yielding $\Pi^* = u-c$.
- if $\mu \geq \frac{4(v-c)}{3}$ then M chooses between $u-c$ and $2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$. We have

$$\begin{aligned} 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) &\geq u-c \\ \sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) &\geq \frac{u-c}{2\mu} \end{aligned}$$

  Clearly, if $\mu \leq 2(u-c)$, then last inequality above cannot hold because the LHS is less than or equal to $\frac{1}{4}$. So if $\frac{4(v-c)}{3} \leq \mu \leq 2(u-c)$, the optimal solution continues to be $f^* = u - c$, yielding $\Pi^* = u - c$. So suppose $\mu > 2(u-c)$. Let $x = \sqrt{1 - \frac{v-c}{\mu}}$ and $y = \frac{u-c}{2\mu} < \frac{1}{4}$. Then the last inequality above is equivalent to

$$\begin{aligned} x^2 - x + y &\leq 0 \\ \frac{1-\sqrt{1-4y}}{2} &\leq x \leq \frac{1+\sqrt{1-4y}}{2} \\ \frac{1-\sqrt{1-\frac{2(u-c)}{\mu}}}{2} &\leq \sqrt{1-\frac{v-c}{\mu}} \leq \frac{1+\sqrt{1-\frac{2(u-c)}{\mu}}}{2} \end{aligned}$$

The LHS inequality always holds when $\mu > 2(u-c) > \frac{4(v-c)}{3}$. Indeed,

$$\frac{1-\sqrt{1-\frac{2(u-c)}{\mu}}}{2} \leq \sqrt{1-\frac{v-c}{\mu}}$$

is equivalent to

$$1 - 2\frac{v-c}{\mu} + \frac{(u-c)}{\mu} + \sqrt{1-\frac{2(u-c)}{\mu}} \geq 0,$$

which holds because

$$1 - 2\frac{v-c}{\mu} + \frac{(u-c)}{\mu} \geq 1 - 2\frac{v-c}{\mu} + \frac{2(v-c)}{3\mu} = 1 - \frac{4(v-c)}{3\mu} > 0.$$

Thus, when $\mu > 2(u-c)$ the inequality

$$\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right) \geq \frac{u-c}{2\mu}$$

is equivalent to

$$\sqrt{1-\frac{v-c}{\mu}} \leq \frac{1+\sqrt{1-\frac{2(u-c)}{\mu}}}{2},$$

i.e.

$$\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$$

And it can be verified that

$$\frac{(2(v-c)-(u-c))^2}{4(v-u)} > 2(u-c).$$

Bottomline for this case is that solution is $f^* = u - c$ and $\Pi^* = u - c$ for $\mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$, and $f^* = 2\mu\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$ and $\Pi^* = 2\mu\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)$

for $\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$.

Suppose $\frac{3(v-c)}{4} < u - c \leq v - c$. Then:

- if $u - c < \mu \leq \frac{4(v-c)}{3}$, the optimal solution is $f^* = u - c$, yielding $\Pi^* = u - c$
- if $\frac{4(v-c)}{3} \leq \mu \leq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$, the optimal solution is $f^* = u - c$, yielding $\Pi^* = u - c$
- if $\mu \geq \frac{(2(v-c)-(u-c))^2}{4(v-u)}$ then M chooses $f^* = 2\mu\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right)$ and

$$\Pi^* = 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right).$$

We have thus proven the expressions of f^* and Π^* given above.

■

Using the expressions from the text of Lemma 4, we now verify that the same results stated in Proposition 7 from the main text continue to hold here.

First, the proof of Lemma 4 has already shown that S_h makes all sales.

Second, it is easily seen that Π^* is weakly increasing in u.

Third, it is easily seen that when $u \to c$, we have

$$\Pi_s^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq \frac{4(v-c)}{3} \\ 2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) & \text{if} \quad \mu \geq \frac{4(v-c)}{3} \end{cases}.$$

This is the profit with a single seller of value v (the low-quality seller with value u is irrelevant when $u \to c$), assuming M can steer and hides the seller when it sets $p_m > v$. Note the difference with the baseline model in the paper, where we have assumed no steering or, if steering is possible, that M shows the seller when $p_m > v$ (in that case, M is indifferent between showing and not showing the seller). The monopoly profit in the baseline was

$$\Pi_{ns}^* = \begin{cases} \frac{\mu}{2} & \text{if} \quad \mu \leq v - c \\ (v-c)\left(1 - \frac{v-c}{2\mu}\right) & \text{if} \quad \mu \geq v - c \end{cases}.$$

Comparing the two profit expressions, we have $\Pi_s^* \geq \Pi_{ns}^*$ for all μ. To see this, note that $(v-c)\left(1 - \frac{v-c}{2\mu}\right) \leq \frac{\mu}{2}$ for all μ and

$$2\mu\sqrt{1 - \frac{v-c}{\mu}}\left(1 - \sqrt{1 - \frac{v-c}{\mu}}\right) > (v-c)\left(1 - \frac{v-c}{2\mu}\right)$$

is equivalent to

$$\sqrt{1-\frac{v-c}{\mu}}\left(1-\sqrt{1-\frac{v-c}{\mu}}\right)>\left(1-\frac{v-c}{2\mu}\right)\frac{v-c}{2\mu}.$$

The last inequality is true when $\mu \geq \frac{4(v-c)}{3}$ because in that case

$$1-\frac{v-c}{2\mu}>\sqrt{1-\frac{v-c}{\mu}}\geq\frac{1}{2}.$$

Thus, since the profit expression in Lemma 4 is increasing in u and equal to Π_s^* when $u \to c$, while the profit with two sellers and no steering from Lemma 2 is decreasing in u and equal to Π_{ns}^* when $u \to c$, we can conclude that here too, M's profit with steering is always higher than M's profit without steering.

E Competing sellers with low-quality seller only active on the marketplace

Here we assume the low-quality seller (whose product offers utility u) does not have a direct channel so is only active on M. The high-quality seller is still active in both channels.

For the case without steering, we prove the following result.

Lemma 5 If the low-quality seller is only active on M and M does not (or cannot) steer, then M obtains the exact same profits as in the baseline, i.e. in the absence of the low-quality seller.

Proof of Lemma 5

Using the same reasoning as in the proof of Proposition 6 in the main text, the high-quality seller (S_h) must make all the sales on and off M in equilibrium. Given this, the low-quality seller (S_l)'s price on M is $p_m^l = c + f$ (this is the only price it sets). Again, by similar arguments as in the proof of Proposition 6, $f \leq v - c$ and $p_d^h \leq p_m^h$. And we must also have $p_m^h \leq \min\{c+f+v-u, v\}$ or $p_d^h \leq v$, which implies either $p_m^h = \min\{c+f+v-u, v\}$ or $p_d^h = v$. It is then easily verified that here too we must always have $p_m^h = \min\{c+f+v-u, v\}$.

There are then two cases:

a) If $f \leq u - c$, then $p_m^h = c + f + v - u$ and S_h solves

$$\max_{p_d^h \leq c+f+v-u}\left\{\left(p_d^h-c\right)\frac{\min\left\{c+f+v-u-p_d^h,\mu\right\}}{\mu}+(v-u)\left(\frac{\mu-\min\left\{c+f+v-u-p_d^h,\mu\right\}}{\mu}\right)\right\},$$

where the $p_d^h \le c+f+v-u$ constraint comes from $p_d^h \le p_m^h$. It is easily verified that the solution is $p_d^{h*} = c+\frac{f}{2}+v-u+\max\left\{0, \frac{f}{2}-\mu\right\}$, so M's profits in this case are $f\left(1-\min\left\{\frac{f}{2\mu}, 1\right\}\right)$.

b) If $f > u-c$, then $p_m^h = v$ and S_h solves

$$\max_{p_d^h \le v}\left\{\left(p_d^h - c\right)\frac{\min\left\{v-p_d^h, \mu\right\}}{\mu} + (v-c-f)\left(\frac{\mu - \min\left\{v-p_d^h, \mu\right\}}{\mu}\right)\right\}$$

It is easily verified the solution is $p_d^{h*} = v - \min\left\{\frac{f}{2}, \mu\right\}$, so M's profits in this case are once again $f\left(1-\min\left\{\frac{f}{2\mu}, 1\right\}\right)$.

Thus, in all cases M makes $f\left(1-\min\left\{\frac{f}{2\mu}, 1\right\}\right)$, which is the exact same profit (as a function of f) as in the baseline, i.e. when the low-quality seller was absent. Optimizing over f leads to the same solution for M.

■

Thus, the presence of a low-quality seller without a direct channel has no impact on leakage and marketplace profits when steering is not possible. To understand this, note that in case b) above, $f > u-c$ renders the low-quality seller irrelevant. Meanwhile, in case a), the low-quality seller does constrain the high-quality seller's pricing on M, but because the high-quality seller is a monopolist in the direct channel, it can adjust its direct price downwards, so the amount of leakage is independent of u. Indeed, note that $p_m^h - p_d^h = \min\left\{\frac{f}{2}, \mu\right\}$, which is exactly the same as in the baseline. This means that the high-quality seller makes lower profits than in the baseline model, but induces the same amount of leakage.

Consider now the case when M can steer. We make the same assumptions about M's steering decision as in the main text: given a set of prices chosen by the two sellers, M shows the seller that induces the least amount of leakage subject to offering non-negative utility to buyers via M, and when indifferent, it shows the high-quality seller.

We first prove the following result.

Lemma 6 If the low-quality seller is only active on M and M does not (or cannot) steer, then M's profits are

$$\Pi^* = \begin{cases} u-c & \text{if} \quad \mu \le u-c \\ \frac{\mu}{2} & \text{if} \quad u-c < \mu \le v-c \\ (v-c)\left(1-\frac{v-c}{2\mu}\right) & \text{if} \quad \mu > v-c \end{cases}. \tag{18}$$

Proof of Lemma 6

Again, the high-quality seller makes all sales on and off M (similar argument to that in the proof of Proposition 7 in the main text, but simpler). Given that S_l does not have a direct channel, everything is as if it had one but chose to set $p_m^l = p_d^l$. Furthermore, there is no reason for S_l to set $p_m^l < u$, so S_l sets $p_m^l = \max\{u, c + f\}$.

If $f > u - c$, then S_l is irrelevant, so everything is as in the baseline, meaning M's profit is $f\left(1 - \min\left\{\frac{f}{2\mu}, 1\right\}\right)$ provided $f \leq v - c$.

If $f \leq u - c$, then S_l is relevant and induces no leakage. Thus, in order to be recommended, S_h must induce no leakage either, meaning we must have $p_d^h = p_m^h = v$. This means M will recommend S_h and make profits equal to f.

So M's profits as a function of f are

$$\mathbf{\Pi}(f) = \begin{cases} f & \text{if} & f \leq u - c \\ f\left(1 - \frac{f}{2\mu}\right) & \text{if} & u - c < f \leq \min\{2\mu, v - c\} \\ 0 & \text{if} & f > 2\mu \end{cases}$$

Optimizing over f, it is easily seen that we obtain the profit expression Π^* given in the text of the Lemma.

■

First, note that the profit expression Π^* given in (18) is increasing in u, which confirms that even without a direct channel, a more competitive low-quality seller is better for M when it can steer.

Second, comparing with M's profit in the baseline given by (6), it is apparent that the two profits are equal except in the range $\mu \leq u - c$, where the profit with a competing low-quality seller without a direct channel is strictly higher. So adding the low-quality seller is weakly better for M.

Finally, it can be easily verified that M's profit with steering when the low-quality seller does not have a direct channel (18) is weakly lower than M's profit with steering when the low-quality seller has a direct channel (16) and (15).