Resale price maintenance and minimum requirements: Online Appendix

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This online appendix provides the proofs behind some of the results in the main paper. It includes the proofs that are similar to the existing proofs in the main paper (specifically, the proof of the second part of Proposition 6 and the proof of Proposition 8). It also provides the proofs of the results in the extensions (Section 7), where we analyze variants of our model with general demand and cost functions, with revenue sharing, and with private benefits as the source of private information.

A Proof of Proposition 6

If $G(.)$ is uniform on $\left[\overline{\theta}-\sigma,\overline{\theta}+\sigma\right]$, then $G(\theta)=\frac{\theta-\overline{\theta}+\sigma}{2\sigma}$. We first provide the proof for the case $\beta>\phi^2$ (upward bias).

If $\beta>\phi^2$, the principal sets a maximum threshold x, so

$$\begin{aligned}\Pi^H(w,x) &= \int_{\overline{\theta}-\sigma}^{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)}\frac{\left(\theta+w\Phi^2\right)^2-w^2\left(\beta-\phi^2\right)^2}{4\sigma\left(2\beta-\phi^2\right)}d\theta \\ &+\int_{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)}^{\overline{\theta}+\sigma}\left(x\left(\theta-\beta x+w\Phi^2\right)+\frac{\phi^2}{2}x^2\right)\frac{d\theta}{2\sigma}-\frac{\Phi^2+\phi^2}{2}w^2.\end{aligned}$$

The respective first-order conditions in x and w can be written

$$\frac{1}{4\sigma}\left(\overline{\theta}+\sigma+w\left(\Phi^2+\beta-\phi^2\right)-\left(2\beta-\phi^2\right)x\right)\left(\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)-\left(2\beta-\phi^2\right)x\right)=0 \tag{A.1}$$

and

$$\int_{\overline{\theta}-\sigma}^{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)}\frac{\Phi^2\left(\theta+w\Phi^2\right)-w\left(\beta-\phi^2\right)^2}{2\sigma\left(2\beta-\phi^2\right)}d\theta+\int_{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)}^{\overline{\theta}+\sigma}x\Phi^2\frac{d\theta}{2\sigma}-\left(\Phi^2+\phi^2\right)w=0. \tag{A.2}$$

Suppose that the (w^*,x^*) which maximizes Π^H is interior, i.e.[1]

$$\frac{\overline{\theta}-\sigma+w^*\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}<x^*<\frac{\overline{\theta}+\sigma+w^*\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}. \tag{A.3}$$

Then (w^*,x^*) is the solution to (A.1) and (A.2). The right-hand side inequality in (A.3) and (A.1) imply that the second-order condition in x holds and that we must have

$$x^*=\frac{\overline{\theta}+\sigma+w^*\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}. \tag{A.4}$$

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[1] Recall indeed that $p^A(w,\theta)=\frac{\theta+w\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}$.

Given that $\beta > \phi^2$, the left-hand side inequality in (A.3) is then equivalent to $w^* < \frac{\sigma}{\beta - \phi^2}$.

Substituting (A.4) into (A.2) and simplifying, we obtain that w^* must be a solution to

$$\Phi^2 \sigma \overline{\theta} - w\sigma \left(\beta^2 - \Phi^4 + 2\beta\Phi^2 - \Phi^2\phi^2\right) + w^2 \left(\beta - \phi^2\right)^3 = 0 \tag{A.5}$$

This quadratic equation has real solutions if and only if $\sigma \geq \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2)^2}$. If $\sigma < \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2)^2}$, the function $f(w) \equiv \Pi^H \left(w, x = \frac{\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}\right)$ is weakly increasing in w for all $w < \frac{\sigma}{\beta-\phi^2}$, so the optimal (w^*, x^*) is non-interior and therefore the H-mode is dominated by either the P-mode or the A-mode.

Assume now $\sigma \geq \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2)^2}$ and denote the two solutions to (A.5) by

$$\begin{aligned} w_1 &= \frac{\sigma}{2\left(\beta-\phi^2\right)^3}\left(\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right) - \sqrt{\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right)^2 - \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{\sigma}}\right) \\ w_2 &= \frac{\sigma}{2\left(\beta-\phi^2\right)^3}\left(\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right) + \sqrt{\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right)^2 - \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{\sigma}}\right) \end{aligned}$$

In this case, the function $f(w)$ is increasing for $w \in [0, w_1]$, decreasing for $w \in [w_1, w_2]$ and increasing again for $w \geq w_2$. Thus, the only candidate interior solution is $w^{H*} = w_1$. This solution is indeed interior if and only if $w_1 < \frac{\sigma}{\beta-\phi^2}$, which is equivalent to

$$\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right) - 2\left(\beta-\phi^2\right)^2 < \sqrt{\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right)^2 - \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{\sigma}}$$

The last inequality is equivalent to

$$\begin{aligned} \left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4 &< \left(\beta-\phi^2\right)^2 \\ &\text{or} \\ \left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4 &\geq \left(\beta-\phi^2\right)^2 \text{ and } \sigma > \frac{\Phi^2\overline{\theta}\left(\beta-\phi^2\right)}{\left(2\beta-\phi^2\right)\left(\Phi^2+\phi^2\right)-\Phi^4}. \end{aligned}$$

Combining this with the requirement that $\sigma \geq \frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2)^2}$ and noting that

$$\frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right)^2} < \frac{\Phi^2\overline{\theta}\left(\beta-\phi^2\right)}{\left(2\beta-\phi^2\right)\left(\Phi^2+\phi^2\right)-\Phi^4}$$

for all parameter values, we obtain that the optimal solution in H-mode is interior if and only if

$$\begin{aligned} \sigma &\in \left[\frac{4\Phi^2\overline{\theta}\left(\beta-\phi^2\right)^3}{\left(\beta^2-\Phi^4+2\beta\Phi^2-\Phi^2\phi^2\right)^2}, \frac{\Phi^2\overline{\theta}\left(\beta-\phi^2\right)}{\left(2\beta-\phi^2\right)\left(\Phi^2+\phi^2\right)-\Phi^4}\right] \text{ and } \left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4 < \left(\beta-\phi^2\right)^2 \\ &\text{or} \\ \sigma &> \frac{\Phi^2\overline{\theta}\left(\beta-\phi^2\right)}{\left(2\beta-\phi^2\right)\left(\Phi^2+\phi^2\right)-\Phi^4}. \end{aligned}$$

If the optimal H-solution is not interior, then the H-mode is dominated by the A-mode or the P-mode.

Finally, let us determine the effect of σ on profits. We have

$$\Pi^{H*} = \max_{w,x} \Pi^H(w,x) = \max_{w} \Pi^H\left(w, x = \frac{\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}\right)$$

We then have

$$\Pi^H\left(w, \frac{\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}\right) = \Pi^H\left(w, \frac{\overline{\theta}+\sigma+w\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}\right) - \int_{\frac{\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}}^{\frac{\overline{\theta}+\sigma+w\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}} \frac{\partial \Pi^H(w,x)}{\partial x} dx$$

$$\begin{aligned}
&= \int_{\overline{\theta}-\sigma}^{\overline{\theta}+\sigma} \frac{\left(\theta+w\Phi^2\right)^2 - w^2\left(\beta-\phi^2\right)^2}{4\sigma\left(2\beta-\phi^2\right)} d\theta - \frac{\Phi^2+\phi^2}{2} w^2 \\
&\quad - \int_{\frac{\overline{\theta}+\sigma+w\left(\Phi^2-\beta+\phi^2\right)}{2\beta-\phi^2}}^{\frac{\overline{\theta}+\sigma+w\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}} \frac{1}{4\sigma}\left(\left(\overline{\theta}+\sigma+w\Phi^2-\left(2\beta-\phi^2\right)x\right)^2 - w^2\left(\beta-\phi^2\right)^2\right) dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{\left(\overline{\theta}+w\Phi^2\right)^2 - w^2\left(\beta-\phi^2\right)^2}{2\left(2\beta-\phi^2\right)} - \frac{\Phi^2+\phi^2}{2} w^2 + \frac{\sigma^2}{6\left(2\beta-\phi^2\right)} + \frac{w^3\left(\beta-\phi^2\right)^3}{3\sigma\left(2\beta-\phi^2\right)} \\
&\equiv \widetilde{\Pi}^H(w,\sigma)
\end{aligned}$$

Using the envelope theorem, we obtain

$$\frac{d\Pi^{H*}}{d\sigma} = \frac{\partial \widetilde{\Pi}^H\left(w = w^{H*}, \sigma\right)}{\partial \sigma} = \frac{\sigma^3 - (w^*)^3\left(\beta-\phi^2\right)^3}{3\sigma^2\left(2\beta-\phi^2\right)} > 0$$

Since $\frac{d\Pi^{P*}}{d\sigma} = 0$ and $\frac{d\Pi^{A*}}{d\sigma} = \frac{\sigma}{3(2\beta-\phi^2)}$, we can conclude $\frac{d\Pi^{P*}}{d\sigma} < \frac{d\Pi^{H*}}{d\sigma} < \frac{d\Pi^{H*}}{d\sigma}$.

This completes the proof for the case $\beta > \phi^2$ (upward bias).

Now suppose $\beta < \phi^2$, so the agent has a downward bias and the principal sets a minimum threshold. In this case,

$$\begin{aligned}
\Pi^H(w,x) &= \int_{(2\beta-\phi^2)x-w(\Phi^2-\phi^2+\beta)}^{\overline{\theta}+\sigma} \frac{\left(\theta+w\Phi^2\right)^2 - w^2\left(\phi^2-\beta\right)^2}{4\sigma\left(2\beta-\phi^2\right)} d\theta \\
&\quad + \int_{\overline{\theta}-\sigma}^{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)} \left(x\left(\theta-\beta x+w\Phi^2\right)+\frac{\phi^2}{2}x^2\right)\frac{d\theta}{2\sigma} - \frac{\Phi^2+\phi^2}{2}w^2.
\end{aligned}$$

The first-order conditions in x and w are

$$\frac{1}{4\sigma}\left(\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)-\left(2\beta-\phi^2\right)x\right)\left(\left(2\beta-\phi^2\right)x-\left(\overline{\theta}-\sigma\right)-w\left(\Phi^2-\phi^2+\beta\right)\right) = 0 \tag{A.6}$$

and

$$\int_{(2\beta-\phi^2)x-w(\Phi^2-\phi^2+\beta)}^{\overline{\theta}+\sigma} \frac{\Phi^2\left(\theta+w\Phi^2\right)-w\left(\beta-\phi^2\right)^2}{2\sigma\left(2\beta-\phi^2\right)} d\theta + \int_{\overline{\theta}-\sigma}^{\left(2\beta-\phi^2\right)x-w\left(\Phi^2-\phi^2+\beta\right)} x\Phi^2\frac{d\theta}{2\sigma} - \left(\Phi^2+\phi^2\right)w = 0. \tag{A.7}$$

Suppose that the (w^*, x^*) which maximizes Π^H is interior, i.e.[2]

$$\frac{\overline{\theta}-\sigma+w^*\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}<x^*<\frac{\overline{\theta}+\sigma+w^*\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}. \tag{A.8}$$

Then (x^*, w^*) is the solution to (A.6) and (A.7). The left-hand side inequality in (A.8) and (A.6) imply that we must have

$$x^*=\frac{\overline{\theta}-\sigma+w^*\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2} \tag{A.9}$$

Given that $\beta<\phi^2$, the right-hand side inequality in (A.8) is then equivalent to $w^*<\frac{\sigma}{\phi^2-\beta}$.

Substituting (A.9) into (A.7), we obtain that w^* must be a solution to

$$\sigma\Phi^2\overline{\theta}-w\sigma\left(\left(2\beta-\phi^2\right)\left(\Phi^2+\phi^2\right)+\left(\phi^2-\beta\right)^2-\Phi^4\right)+w^2\left(\phi^2-\beta\right)^3=0 \tag{A.10}$$

This quadratic equation has real solutions if and only if $\sigma\geq\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4)^2}$. If $\sigma<\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4)^2}$, the function $f(w)\equiv\Pi^H\left(w,x=\frac{\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2}\right)$ is weakly increasing in w for all $w<\frac{\sigma}{\phi^2-\beta}$, so the optimal (w^*, x^*) is non-interior and therefore the H-mode is dominated by either the P-mode or the A-mode.

Assume now $\sigma\geq\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4)^2}$ and denote the two solutions to (A.5) by

$$\begin{aligned}
w_1 &= \frac{\sigma}{2\left(\phi^2-\beta\right)^3}\left(\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)-\sqrt{\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)^2-\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{\sigma}}\right)\\
w_2 &= \frac{\sigma}{2\left(\phi^2-\beta\right)^3}\left(\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)+\sqrt{\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)^2-\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{\sigma}}\right)
\end{aligned}$$

In this case, the function $f(w)$ is increasing for $w\in[0,w_1]$, decreasing for $w\in[w_1,w_2]$ and increasing again for $w\geq w_2$. Thus, the only candidate interior solution is $w^{H*}=w_1$. This solution is indeed interior if and only if $w_1<\frac{\sigma}{\phi^2-\beta}$, which is equivalent to

$$\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)-2\left(\phi^2-\beta\right)^2<\sqrt{\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)^2-\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{\sigma}}$$

The last inequality is equivalent to

$$\begin{aligned}
\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4 &< \left(\phi^2-\beta\right)^2\\
&\text{or}\\
\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4 &\geq \left(\phi^2-\beta\right)^2 \text{ and } \sigma>\frac{\Phi^2\overline{\theta}\left(\phi^2-\beta\right)}{\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4}.
\end{aligned}$$

Combining this with the requirement that $\sigma\geq\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4)^2}$ and noting that

$$\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)^2}<\frac{\Phi^2\overline{\theta}\left(\phi^2-\beta\right)}{\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4}$$

[2]Recall indeed that $p^A(w,\theta)=\frac{\theta+w\left(\Phi^2+\beta-\phi^2\right)}{2\beta-\phi^2}$.

for all parameter values, we obtain that the optimal solution in H-mode is interior if and only if

$$\sigma \in \left[\frac{4\Phi^2\overline{\theta}\left(\phi^2-\beta\right)^3}{\left(\beta^2+2\beta\Phi^2-\Phi^2\phi^2-\Phi^4\right)^2}, \frac{\Phi^2\overline{\theta}\left(\phi^2-\beta\right)}{\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4}\right] \text{ and } \left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4<\left(\phi^2-\beta\right)^2$$

or

$$\sigma > \frac{\Phi^2\overline{\theta}\left(\phi^2-\beta\right)}{\left(2\beta-\phi^2\right)\left(\phi^2+\Phi^2\right)-\Phi^4}.$$

If the optimal H-solution is not interior, then the H-mode is dominated by the A-mode or the P-mode.

Finally, let us determine the effect of σ on profits. We have

$$\Pi^{H*}=\max_{w,x}\Pi^H(w,x)=\max_{w}\Pi^H\left(w, x=\frac{\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2}\right)$$

We then have

$$\Pi^H\left(w,\frac{\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2}\right)=\Pi^H\left(w,\frac{\overline{\theta}-\sigma+w\left(\Phi^2-\phi^2+\beta\right)}{2\beta-\phi^2}\right)+\int_{\frac{\overline{\theta}-\sigma+w\left(\Phi^2-\phi^2+\beta\right)}{2\beta-\phi^2}}^{\frac{\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2}}\frac{\partial\Pi^H(w,x)}{\partial x}dx$$

$$\begin{aligned}
&= \int_{\overline{\theta}-\sigma}^{\overline{\theta}+\sigma}\frac{\left(\theta+w\Phi^2\right)^2-w^2\left(\phi^2-\beta\right)^2}{4\sigma\left(2\beta-\phi^2\right)}d\theta-\frac{\Phi^2+\phi^2}{2}w^2 \\
&\quad+\int_{\frac{\overline{\theta}-\sigma+w\left(\Phi^2-\phi^2+\beta\right)}{2\beta-\phi^2}}^{\frac{\overline{\theta}-\sigma+w\left(\Phi^2+\phi^2-\beta\right)}{2\beta-\phi^2}}\frac{1}{4\sigma}\left(w^2\left(\phi^2-\beta\right)^2-\left(\overline{\theta}-\sigma+w\Phi^2-\left(2\beta-\phi^2\right)x\right)^2\right)dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{\left(\overline{\theta}+w\Phi^2\right)^2-w^2\left(\phi^2-\beta\right)^2}{2\left(2\beta-\phi^2\right)}-\frac{\Phi^2+\phi^2}{2}w^2+\frac{\sigma^2}{6\left(2\beta-\phi^2\right)}+\frac{w^3\left(\phi^2-\beta\right)^3}{3\sigma\left(2\beta-\phi^2\right)} \\
&\equiv \widetilde{\Pi}^H(w,\sigma)
\end{aligned}$$

So, using the envelope theorem, we get

$$\frac{d\Pi^{H*}}{d\sigma}=\frac{\partial\widetilde{\Pi}^H\left(w=w^{H*},\sigma\right)}{\partial\sigma}=\frac{\sigma^3-\left(w^{H*}\right)^3\left(\phi^2-\beta\right)^3}{3\sigma^2\left(2\beta-\phi^2\right)}>0$$

Since $\frac{d\Pi^{P*}}{d\sigma}=0$ and $\frac{d\Pi^{A*}}{d\sigma}=\frac{\sigma}{3(2\beta-\phi^2)}$, we can conclude $\frac{d\Pi^{P*}}{d\sigma}<\frac{d\Pi^{H*}}{d\sigma}<\frac{d\Pi^{H*}}{d\sigma}$.

B Proof of Proposition 8

Recall that

$$\begin{aligned}\Pi^{H*} &= \max_{w,x}\left\{\mathbb{E}\left[p\left(\theta-\beta p+\phi q+\Phi Q\right)-\frac{1}{2}q^2-\frac{1}{2}Q^2\right]\right\}\\ &\quad\text{subject to}\\ p &= \frac{1}{2}w+\frac{1}{2\beta}\left(\theta+\phi q+w\Phi^2\right)\\ q &= \max\left\{q^A\left(w,\theta\right),x\right\}\\ Q &= w\Phi.\end{aligned}$$

Plugging these constraints into the principal's objective function, we obtain $\Pi^{H*}=\max_{w,x}\Pi^H\left(w,x\right)$, where

$$\Pi^H\left(w,x\right)\equiv\int_{\theta_L}^{\frac{2\beta-\phi^2}{\phi}x+w\left(\beta-\Phi^2\right)}\left(\frac{1}{4\beta}\left(\theta+\phi x+w\left(\Phi^2-\beta\right)\right)^2+\frac{w}{2}\left(\theta+\phi x-\beta w\right)-\frac{1}{2}x^2\right)dG\left(\theta\right)$$

$$+\int_{\frac{2\beta-\phi^2}{\phi}x+w\left(\beta-\Phi^2\right)}^{\theta_H}\left(\frac{1}{4\beta}\left(\theta+\phi q^A\left(w,\theta\right)+w\left(\Phi^2-\beta\right)\right)^2+\frac{w}{2}\left(\theta+\phi q^A\left(w,\theta\right)-\beta w\right)-\frac{1}{2}q^A\left(w,\theta\right)^2\right)dG\left(\theta\right).$$

Compare then the A-mode to the H-mode with $w^H=w^{A*}$ and $x=q^A\left(w^{A*},\theta_L+\kappa\right)=\frac{\left(\theta_L+\kappa+w\left(\Phi^2-\beta\right)\right)\phi}{2\beta-\phi^2}$, where $\kappa>0$ is small. We can then write

$$\Pi^H\left(w^{A*},x=q^A\left(w^{A*},\theta_L+\kappa\right)\right)-\Pi^{A*}=\Pi^H\left(w^{A*},x=q^A\left(w^{A*},\theta_L+\kappa\right)\right)-\Pi^H\left(w^{A*},x=q^A\left(w^{A*},\theta_L\right)\right)$$

$$\begin{aligned}&=\int_{\theta_L}^{\theta_L+\kappa}\frac{1}{4\beta}\left(q^A\left(w^{A*},\theta_L+\kappa\right)-q^A\left(w^{A*},\theta\right)\right)\begin{pmatrix}2\theta\phi+2w^{A*}\Phi^2\phi\\ -\left(2\beta-\phi^2\right)\left(q^A\left(w^{A*},\theta_L+\kappa\right)+q^A\left(w^{A*},\theta\right)\right)\end{pmatrix}dG\left(\theta\right)\\ &=\int_{\theta_L}^{\theta_L+\kappa}\frac{\phi^2}{4\beta\left(2\beta-\phi^2\right)}\left(\theta_L+\kappa-\theta\right)\left(\theta-\theta_L-\kappa+2w^{A*}\beta\right)dG\left(\theta\right).\end{aligned}$$

The last expression is positive for κ sufficiently small because $w^{A*}>0$. Thus, the H-mode dominates the A-mode.

Next, compare the P-mode to the H-mode with $w^H=w^{P*}$ and $x=q^{P*}=\frac{\left(\overline{\theta}+w^{P*}\Phi^2\right)\phi}{2\beta-\phi^2}$. Note that this $\left(w^H,x\right)$ is strictly interior because

$$\theta_L<\frac{2\beta-\phi^2}{\phi}q^{P*}+w^{P*}\left(\beta-\Phi^2\right)=\overline{\theta}+w^{P*}\beta<\theta_H,$$

where the second inequality follows from the assumption $\frac{\theta_H}{\overline{\theta}}>1+\frac{\Phi^2\beta}{\left(2\beta-\phi^2\right)\left(\frac{\beta}{2}+\Phi^2\right)-2\Phi^4}$. We can then write

$$\Pi^H\left(w^{P*},x=q^{P*}\right)-\Pi^{P*}=\Pi^H\left(w^{P*},q^{P*}\right)-\Pi^P\left(w^{P*},q^{P*}\right)$$

$$=\int_{\overline{\theta}+w^{P*}\beta}^{\theta_H}\begin{pmatrix}\frac{1}{4\beta}\left(\theta+\phi q^A\left(w^{P*},\theta\right)+w^{P*}\left(\Phi^2-\beta\right)\right)^2+\frac{w^{P*}}{2}\left(\theta+\phi q^A\left(w^{P*},\theta\right)-\beta w^{P*}\right)-\frac{1}{2}q^A\left(w^{P*},\theta\right)^2\\ -\left(\frac{1}{4\beta}\left(\theta+\phi q^{P*}+w^{P*}\left(\Phi^2-\beta\right)\right)^2+\frac{w^{P*}}{2}\left(\theta+\phi q^{P*}-\beta w^{P*}\right)-\frac{1}{2}\left(q^{P*}\right)^2\right)\end{pmatrix}dG\left(\theta\right)$$

$$=\int_{\overline{\theta}+w^{P*}\beta}^{\theta_H}\frac{1}{4\beta}\left(q^A\left(w^{P*},\theta\right)-q^{P*}\right)\left(2\theta\phi+2w^{P*}\Phi^2\phi-\left(2\beta-\phi^2\right)\left(q^A\left(w^{P*},\theta\right)+q^{P*}\right)\right)dG\left(\theta\right)$$

$$= \int_{\overline{\theta}+w^{P*}\beta}^{\theta_H} \frac{\phi^2}{4\beta\left(2\beta-\phi^2\right)} \left(\theta-\overline{\theta}-w^{P*}\beta\right)\left(\theta-\overline{\theta}+w^{P*}\beta\right) dG\left(\theta\right) > 0.$$

Thus, the H-mode dominates the P-mode.

C General demand and cost functions

We focus on the case when the agent has an upward bias at $w = w^{P*}$ and $w = w^{A*}$, i.e. $p^A\left(w^{P*},\theta\right) > p^P\left(w^{P*},\theta\right)$ and $p^A\left(w^{A*},\theta\right) > p^P\left(w^{A*},\theta\right)$ for all θ. For this case, consider the H-mode with a maximum threshold on p:

$$\begin{aligned} & \max_{w,x} \left\{\mathbb{E}\left[pD\left(\theta,p,q,Q\right)-c\left(q\right)-C\left(Q\right)\right]\right\} \\ & \text{subject to} \\ p &= \min\left\{p^A\left(w,\theta\right),x\right\} \\ q &= q\left(p,w,\theta\right) \\ Q &= Q\left(p,w\right) \end{aligned}$$

where the functions $Q\left(p,w\right)$, $q\left(p,w,\theta\right)$ and $p^A\left(w,\theta\right)$ are as defined in Section 7.1 in the main text.

Compare A-mode to H-mode with $\left(w=w^{A*},x=p^A\left(w^{A*},\theta_H-\kappa\right)\right)$ for small κ:

$$\begin{aligned} & \Pi^H\left(w^{A*},x=p^A\left(w^{A*},\theta_H-\kappa\right)\right)-\Pi^{A*} = \Pi^H\left(w^{A*},x=p^A\left(w^{A*},\theta_H-\kappa\right)\right)-\Pi^H\left(w^{A*},x=p^A\left(w^{A*},\theta_H\right)\right) \\ = & \int_{\theta_H-\kappa}^{\theta_H} y\left(\theta,\kappa\right) dG\left(\theta\right), \end{aligned}$$

where

$$\begin{aligned} y\left(\theta,\kappa\right) \equiv {} & p^A\left(w^{A*},\theta_H-\kappa\right) D\left(\theta,p^A\left(w^{A*},\theta_H-\kappa\right),q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*}\right)\right) \\ & -c\left(q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right)\right)-C\left(Q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*}\right)\right) \\ & -\left(\begin{array}{c} p^A\left(w^{A*},\theta\right) D\left(\theta,p^A\left(w^{A*},\theta\right),q\left(p^A\left(w^{A*},\theta\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta\right),w^{A*}\right)\right) \\ -c\left(q\left(p^A\left(w^{A*},\theta\right),w^{A*},\theta\right)\right)-C\left(Q\left(p^A\left(w^{A*},\theta\right),w^{A*}\right)\right) \end{array}\right). \end{aligned}$$

We have

$$\frac{\partial y\left(\theta,\kappa\right)}{\partial\theta} = \left(\begin{array}{c} p^A\left(w^{A*},\theta_H-\kappa\right) \frac{\partial D\left(\theta,p^A\left(w^{A*},\theta_H-\kappa\right),q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*}\right)\right)}{\partial\theta} \\ -p^A\left(w^{A*},\theta\right) \frac{\partial D\left(\theta,p^A\left(w^{A*},\theta\right),q\left(p^A\left(w^{A*},\theta\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta\right),w^{A*}\right)\right)}{\partial\theta} \end{array}\right)$$

$$+\left(\begin{array}{c} \left(\begin{array}{c} p^A\left(w^{A*},\theta_H-\kappa\right) \frac{\partial D\left(\theta,p^A\left(w^{A*},\theta_H-\kappa\right),q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*}\right)\right)}{\partial q} \\ -c'\left(q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right)\right) \end{array}\right) \frac{\partial q\left(p^A\left(w^{A*},\theta_H-\kappa\right),w^{A*},\theta\right)}{\partial\theta} \\ -\left(\begin{array}{c} p^A\left(w^{A*},\theta\right) \frac{\partial D\left(\theta,p^A\left(w^{A*},\theta\right),q\left(p^A\left(w^{A*},\theta\right),w^{A*},\theta\right),Q\left(p^A\left(w^{A*},\theta\right),w^{A*}\right)\right)}{\partial\theta} \\ -c'\left(q\left(p^A\left(w^{A*},\theta\right),w^{A*},\theta\right)\right) \end{array}\right) \frac{\partial q\left(p^A\left(w^{A*},\theta_H\right),w^{A*},\theta\right)}{\partial\theta} \end{array}\right)$$

$$-\frac{\partial p^A\left(w^{A*},\theta\right)}{\partial\theta} \times \left.\frac{d\left(pD\left(\theta,p,q\left(p,w^{A*},\theta\right),Q\left(p,w^{A*}\right)\right)-c\left(q\left(p,w^{A*},\theta\right)\right)-C\left(Q\left(p,w^{A*}\right)\right)\right)}{dp}\right|_{p=p^A\left(w^{A*},\theta\right)}.$$

The first two terms in the expression of $\frac{\partial y(\theta,\kappa)}{\partial\theta}$ above vanish as $\kappa\to 0$. Meanwhile, $\frac{\partial p^A\left(w^{A*},\theta\right)}{\partial\theta} > 0$ by assumption and

$$\left.\frac{d\left(pD\left(\theta,p,q\left(p,w^{A*},\theta\right),Q\left(p,w^{A*},\theta\right)\right)-c\left(q\left(p,w^{A*},\theta\right)\right)-C\left(Q\left(p,w^{A*}\right)\right)\right)}{dp}\right|_{p=p^{A}\left(w^{A*},\theta\right)}<0$$

because $p^{A}\left(w^{A*},\theta\right)>p^{P}\left(w^{A*},\theta\right)$ (upward bias),

$$\left.\frac{d\left(pD\left(\theta,p,q\left(p,w^{A*},\theta\right),Q\left(p,w^{A*},\theta\right)\right)-c\left(q\left(p,w^{A*},\theta\right)\right)-C\left(Q\left(p,w^{A*}\right)\right)\right)}{dp}\right|_{p=p^{P}\left(w^{A*},\theta\right)}=0$$

by definition of $p^{P}\left(w^{A*},\theta\right)$ and

$$pD\left(\theta,p,q\left(p,w^{A*},\theta\right),Q\left(p,w^{A*},\theta\right)\right)-c\left(q\left(p,w^{A*},\theta\right)\right)-C\left(Q\left(p,w^{A*}\right)\right)$$

is assumed to be concave in p. Thus, the term

$$\frac{\partial p^{A}\left(w^{A*},\theta\right)}{\partial\theta}\times\left.\frac{d\left(pD\left(\theta,p,q\left(p,w^{A*},\theta\right),Q\left(p,w^{A*},\theta\right)\right)-c\left(q\left(p,w^{A*},\theta\right)\right)-C\left(Q\left(p,w^{A*}\right)\right)\right)}{dp}\right|_{p=p^{A}\left(w^{A*},\theta\right)}$$

is negative for all $\theta\in[\theta_H-\kappa,\theta_H]$ and bounded away from 0 as $\kappa\to 0$. Consequently, we can conclude that if κ is sufficiently small, $\frac{\partial y(\theta,\kappa)}{\partial\theta}>0$ for all $\theta\in[\theta_H-\kappa,\theta_H]$. Furthermore, $y\left(\theta_H-\kappa,\kappa\right)=0$. Thus, if κ is sufficiently small, $y\left(\theta,\kappa\right)>0$ for all $\theta\in\left(\theta_H-\kappa,\theta_H\right)$ and therefore

$$\Pi^{H}\left(w^{A*},x=p^{A}\left(w^{A*},\theta_H-\kappa\right)\right)>\Pi^{A*}.$$

This means the H-mode dominates the A-mode.

Now let us compare P-mode to H-mode with $\left(w^{H}=w^{P*},x=p^{P*}\right)$. If $p^{A}\left(w^{P*},\theta_L\right)<p^{P*}$,[3] then there exists a unique $\theta^{P*}\in\left(\theta_L,\theta_H\right)$ such that $p^{A}\left(w^{P*},\theta^{P*}\right)=p^{P*}$. We can then write

$$\begin{aligned}\Pi^{H}\left(w^{P*},x=p^{P*}\right) &= \int_{\theta_L}^{\theta^{P*}}\left(\begin{array}{c}p^{A}\left(w^{P*},\theta\right)D\left(\theta,p^{A}\left(w^{P*},\theta\right),q\left(p^{A}\left(w^{P*},\theta\right),w^{P*},\theta\right),Q\left(p^{A}\left(w^{P*},\theta\right),w^{P*}\right)\right)\\ -c\left(q\left(p^{A}\left(w^{P*},\theta\right),w^{P*},\theta\right)\right)-C\left(Q\left(p^{A}\left(w^{P*},\theta\right),w^{P*}\right)\right)\end{array}\right)dG\left(\theta\right)\\ &\quad+\int_{\theta^{P*}}^{\theta_H}\left(\begin{array}{c}p^{P*}D\left(\theta,p^{P*},q\left(p^{P*},w^{P*},\theta\right),Q\left(p^{P*},w^{P*}\right)\right)\\ -c\left(q\left(p^{P*},w^{P*},\theta\right)\right)-C\left(Q\left(p^{P*},w^{P*}\right)\right)\end{array}\right)dG\left(\theta\right)\end{aligned}$$

For all $\theta<\theta^{P*}$, we have $p^{P}\left(w^{P*},\theta\right)<p^{A}\left(w^{P*},\theta\right)<p^{P*}$. And since

$$pD\left(\theta,p,q\left(p,w^{P*},\theta\right),Q\left(p,w^{P*}\right)\right)-c\left(q\left(p,w^{P*},\theta\right)\right)-C\left(Q\left(p,w^{P*}\right)\right)$$

[3]Since the agent has an upward bias, we have $p^{A}\left(w^{P*},\theta_H\right)>p^{P}\left(w^{P*},\theta_H\right)\geq p^{P*}$.

is concave in p and maximized by $p = p^P\left(w^{P*}, \theta\right)$, we have

$$\begin{aligned} & p^P\left(w^{P*},\theta\right) D\left(\theta, p^P\left(w^{P*},\theta\right), q\left(p^P\left(w^{P*},\theta\right), w^{P*},\theta\right), Q\left(p^P\left(w^{P*},\theta\right), w^{P*}\right)\right) \\ & -c\left(q\left(p^P\left(w^{P*},\theta\right), w^{P*},\theta\right)\right) - C\left(Q\left(p^P\left(w^{P*},\theta\right), w^{P*}\right)\right) \\ > \; & p^A\left(w^{P*},\theta\right) D\left(\theta, p^A\left(w^{P*},\theta\right), q\left(p^A\left(w^{P*},\theta\right), w^{P*},\theta\right), Q\left(p^A\left(w^{P*},\theta\right), w^{P*}\right)\right) \\ & -c\left(q\left(p^A\left(w^{P*},\theta\right), w^{P*},\theta\right)\right) - C\left(Q\left(p^A\left(w^{P*},\theta\right), w^{P*}\right)\right) \\ > \; & p^{P*} D\left(\theta, p^{P*}, q\left(p^{P*}, w^{P*},\theta\right), Q\left(p^{P*}, w^{P*}\right)\right) \\ & -c\left(q\left(p^{P*}, w^{P*},\theta\right)\right) - C\left(Q\left(p^{P*}, w^{P*}\right)\right). \end{aligned}$$

Thus,

$$\begin{aligned} \Pi^H\left(w^{P*}, x = p^{P*}\right) \; > \; & \int_{\theta_L}^{\theta^{P*}} \begin{pmatrix} p^{P*} D\left(\theta, p^{P*}, q\left(p^{P*}, w^{P*},\theta\right), Q\left(p^{P*}, w^{P*}\right)\right) \\ -c\left(q\left(p^{P*}, w^{P*},\theta\right)\right) - C\left(Q\left(p^{P*}, w^{P*}\right)\right) \end{pmatrix} dG(\theta) \\ & + \int_{\theta^{P*}}^{\theta_H} \begin{pmatrix} p^{P*} D\left(\theta, p^{P*}, q\left(p^{P*}, w^{P*},\theta\right), Q\left(p^{P*}, w^{P*}\right)\right) \\ -c\left(q\left(p^{P*}, w^{P*},\theta\right)\right) - C\left(Q\left(p^{P*}, w^{P*}\right)\right) \end{pmatrix} dG(\theta) \\ = \; & \Pi^{P*} \end{aligned}$$

So H-mode also dominates P-mode.

A symmetric reasoning applies for the case with uniform downward bias, i.e. $p^A(w,\theta) < p^P(w,\theta)$ for all (w,θ).

D Revenue sharing

In this section, we show how our analysis of RPM goes through when the principal charges a revenue share t instead of the wholesale price w, in addition to the fixed fee. Given the timing assumption noted in Section 7.1 of the main paper, we have that p is transferable and set first, while q is non-contractible and always chosen by the agent in the second stage. Let then $(q(p,t,\theta), Q(p,t))$ be the joint solutions to

$$\begin{cases} q = \arg\max_{q'} \left\{(1-t)\, pD(\theta, p, q', Q) - c(q')\right\} \\ Q = \arg\max_{Q'} \left\{\mathbb{E}_\theta\left[tpD(\theta, p, q, Q') - C(Q')\right]\right\} \end{cases}.$$

Also define

$$\begin{aligned} p^P(t,\theta) & \equiv \arg\max_{p'} \left\{p' D\left(\theta, p', q(p', t, \theta), Q(p', t)\right) - c\left(q(p', t, \theta)\right) - C\left(Q(p', t)\right)\right\} \\ p^A(t,\theta) & \equiv \arg\max_{p'} \left\{(1-t)\, p' D\left(\theta, p', q(p', t, \theta), Q(p', t)\right) - c\left(q(p', t, \theta)\right)\right\}. \end{aligned}$$

Thus, $p^P(t,\theta)$ is the optimal price that the principal would like to choose in P-mode for a given t if it could observe θ in the first stage, but not in the second stage when it chooses Q. Meanwhile, $p^A(t,\theta)$ is the price chosen by the agent in A-mode, given t and θ.

We assume that $p^P(t,\theta)$ and $p^A(t,\theta)$ are increasing in θ, consistent with the random term θ being interpeted as a positive demand shock. We also assume that the principal's profit as a function of p,

$$pD\left(\theta, p, q(p,t,\theta), Q(p,t)\right) - c\left(q(p,t,\theta)\right) - C\left(Q(p,t)\right),$$

is concave in p for any (t, θ). Denoting by p^{P*} and t^{P*} the principal's optimal choices of price and revenue share in P-mode and by t^{A*} the principal's optimal choice of revenue share in A-mode, we obtain the corresponding result to Proposition 10 in the main paper (the proof is essentially the same, so we omit here).

Proposition D.1 (Resale price maintenance)
Maximum RPM: *If $p^A\left(t^{A*}, \theta\right) > p^P\left(t^{A*}, \theta\right)$ for all θ (i.e. the agent has an upward bias when the principal charges t^{A*}), then the H-mode with maximum RPM dominates the A-mode. If in addition $p^A\left(t^{P*}, \theta\right) > p^P\left(t^{P*}, \theta\right)$ for all θ (i.e. the agent also has an upward bias when the principal charges t^{P*}) and $p^A\left(t^{P*}, \theta_L\right) < p^{P*}$, then the H-mode with maximum RPM also dominates the P-mode.*

Minimum RPM: If $p^A\left(t^{A*}, \theta\right) < p^P\left(t^{A*}, \theta\right)$ for all θ (i.e. the agent has a downward bias when the principal charges t^{A*}), then the H-mode with minimum RPM dominates the A-mode. If in addition $p^A\left(t^{P*}, \theta\right) < p^P\left(t^{P*}, \theta\right)$ for all θ (i.e. the agent also has a downward bias when the principal charges t^{P*}) and $p^A\left(t^{P*}, \theta_H\right) > p^{P*}$, then the H-mode with minimum RPM also dominates the P-mode.

Now consider the linear-quadratic model of Section 4 but with revenue sharing rather than wholesale prices. We assume

$$2\beta > \phi^2 + 2\Phi^2, \tag{D.1}$$

which ensures second order conditions hold throughout.

Consider first the P-mode. The principal's problem is

$$\begin{aligned} & \max_{t,p} \mathbb{E}_\theta \left\{ p\left(\theta - \beta p + \phi q(t,p) + \Phi Q(t,p)\right) - \frac{1}{2} q(t,p)^2 - \frac{1}{2} Q(t,p)^2 \right\} \\ & \text{subject to} \\ q(t,p) &= (1-t)\, p\phi \\ Q(t,p) &= tp\Phi. \end{aligned}$$

Plugging in the two constraints and taking the respective first-order conditions in t and p, we obtain

$$\begin{aligned} t^{P*} &= \frac{\Phi^2}{\Phi^2 + \phi^2} \\ p^{P*} &= \frac{\overline{\theta}}{2\beta - \left(1 - \left(t^{P*}\right)^2\right)\phi^2 - t^{P*}\left(2 - t^{P*}\right)\Phi^2}. \end{aligned}$$

We also have

$$p^P(t, \theta) = \frac{\theta}{2\beta - (1-t^2)\,\phi^2 - t\,(2-t)\,\Phi^2}.$$

Consider now the A-mode. The price p is set in stage 2a before q and Q are set in stage 2b as in the P-mode. Thus, in stage 2a the agent sets

$$\begin{aligned} p^A(t, \theta) &= \arg\max_p \left\{ (1-t)\, p\left(\theta - \beta p + (1-t)\,\phi^2 p + t\Phi^2 p\right) - \frac{1}{2}(1-t)^2 \phi^2 p^2 \right\} \\ &= \frac{\theta}{2\beta - (1-t)\,\phi^2 - 2t\Phi^2}. \end{aligned}$$

The principal therefore solves

$$\max_{t}\left\{\mathbb{E}\left[p^{A}\left(t,\theta\right)\left(\theta-\left(\beta-\left(1-t\right)\phi^{2}-t\Phi^{2}\right)p^{A}\left(t,\theta\right)\right)-\frac{1}{2}\left(\left(1-t\right)^{2}\phi^{2}+t^{2}\Phi^{2}\right)p^{A}\left(t,\theta\right)^{2}\right]\right\}.$$

We obtain

$$t^{A*}=\frac{\Phi^{2}}{\phi^{2}+\Phi^{2}+\frac{\left(2\Phi^{2}-\phi^{2}\right)\left(\Phi^{2}-\phi^{2}\right)}{2\beta-\phi^{2}}}.$$

Comparing t^{A*} with t^{P*}, we have $t^{A*}<t^{P*}$ if $\left(2\Phi^{2}-\phi^{2}\right)\left(\Phi^{2}-\phi^{2}\right)>0$, and $t^{A*}>t^{P*}$ if $\left(2\Phi^{2}-\phi^{2}\right)\left(\Phi^{2}-\phi^{2}\right)<0$.

Furthermore, $p^{A}\left(t,\theta\right)<p^{P}\left(t,\theta\right)$ (i.e. the agent has downward bias at t) if and only if

$$t<\frac{\phi^{2}}{\Phi^{2}+\phi^{2}}. \tag{D.2}$$

We are interested in the biases at $t=t^{P*}$ and $t=t^{A*}$. We have:

- $p^{A}\left(t^{P*},\theta\right)<p^{P}\left(t^{P*},\theta\right)$ if and only if $\Phi^{2}<\phi^{2}$.
- $p^{A}\left(t^{A*},\theta\right)<p^{P}\left(t^{A*},\theta\right)$ if and only if $\frac{\Phi^{2}}{\phi^{2}+\Phi^{2}+\frac{\left(2\Phi^{2}-\phi^{2}\right)\left(\Phi^{2}-\phi^{2}\right)}{2\beta-\phi^{2}}}<\frac{\phi^{2}}{\Phi^{2}+\phi^{2}}$, which, given $2\beta>\phi^{2}+2\Phi^{2}$, is also equivalent to $\Phi^{2}<\phi^{2}$.

Finally, $p^{A}\left(t^{P*},\theta_{L}\right)<p^{P*}$ is equivalent to

$$\frac{\theta_{L}}{\overline{\theta}}<\frac{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-2\Phi^{4}}{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-\phi^{2}\Phi^{2}-\Phi^{4}}. \tag{D.3}$$

If $\phi^{2}>\Phi^{2}$, then the RHS of (D.3) is larger than 1, so (D.3) is definitely satisfied.

Similarly, $p^{A}\left(t^{P*},\theta_{H}\right)>p^{P*}$ is equivalent to

$$\frac{\theta_{H}}{\overline{\theta}}>\frac{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-2\Phi^{4}}{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-\phi^{2}\Phi^{2}-\Phi^{4}}. \tag{D.4}$$

If $\phi^{2}<\Phi^{2}$, then the RHS of (D.4) is smaller than 1, so (D.4) is definitely satisfied.

Thus, we can conclude that:

- If $\Phi^{2}<\phi^{2}$, then H-mode with minimum RPM dominates A-mode. If in addition $\frac{\theta_{H}}{\overline{\theta}}>\frac{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-2\Phi^{4}}{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-\phi^{2}\Phi^{2}-\Phi^{4}}$, then H-mode with minimum RPM also dominates the P-mode.
- If $\Phi^{2}>\phi^{2}$, then H-mode with maximum RPM dominates A-mode. If in addition $\frac{\theta_{L}}{\overline{\theta}}<\frac{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-2\Phi^{4}}{2\beta\left(\Phi^{2}+\phi^{2}\right)-\phi^{4}-\phi^{2}\Phi^{2}-\Phi^{4}}$, then H-mode with maximum RPM also dominates the P-mode.

E Private benefits

In this section we show that our analysis of RPM in the main paper also applies to the case in which the agent's private information regards his private benefits (or costs). We adopt the same timing as in Section 7.1 of the main paper, so the non-contractible and non-transferable decisions q and Q are always chosen in the second stage after p is determined. Let then $\left(q\left(p,w,b\right),Q\left(p,w\right)\right)$ be the joint solutions to

$$\left\{\begin{array}{c} q=\arg\max_{q'}\left\{\left(p+b-w\right)D\left(p,q',Q\right)-c\left(q'\right)\right\} \\ Q=\arg\max_{Q'}\left\{\mathbb{E}_b\left[wD\left(p,q,Q'\right)-C\left(Q'\right)\right]\right\} \end{array}\right. .$$

Thus, $\left(q\left(p,w,b\right),Q\left(p,w\right)\right)$ is the Nash equilibrium of the game in which the agent and the principal set q and Q respectively, given p and w. Let then

$$\begin{aligned} p^P\left(w,b\right) &\equiv \arg\max_{p'}\left\{\left(p'+b\right)D\left(p',q\left(p',w,b\right),Q\left(p',w\right)\right)-c\left(q\left(p',w,b\right)\right)-C\left(Q\left(p',w\right)\right)\right\} \\ p^A\left(w,b\right) &\equiv \arg\max_{p'}\left\{\left(p'+b-w\right)D\left(p',q\left(p',w,b\right),Q\left(p',w\right)\right)-c\left(q\left(p',w,b\right)\right)\right\}. \end{aligned}$$

We assume that the principal's profit as a function of p,

$$\left(p+b\right)D\left(p,q\left(p,w,b\right),Q\left(p,w\right)\right)-c\left(q\left(p,w,b\right)\right)-C\left(Q\left(p,w\right)\right),$$

is concave in p for any $\left(w,b\right)$. Denoting by p^{P*} and w^{P*} the principal's optimal choices of price and wholesale price in P-mode and by w^{A*} the principal's optimal choice of wholesale price in A-mode, it is easily verified that we obtain the same results as in Proposition 10, replacing θ by b (the proof is the same, so we omit it here).

Now consider the linear-quadratic model of Section 4 but with private benefits rather than a demand shock (so θ is now a constant). Given a wholesale price w and a realization b of the agent's private benefit, the principal's ideal price is

$$p^P\left(w,b\right)=\frac{\theta-b\left(\beta-\phi^2\right)+w\Phi^2}{2\beta-\phi^2},$$

whereas the agent's ideal price is

$$p^A\left(w,b\right)=\frac{\theta-b\left(\beta-\phi^2\right)+w\left(\Phi^2-\phi^2+\beta\right)}{2\beta-\phi^2}.$$

Thus, the agent has a upwards (respectively, downwards) bias if and only if $\beta>\phi^2$.