Controlling versus enabling — Online appendix

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Section 1 of this online appendix contains the proof of the technical Lemma (Lemma 2) used in the Proof of Lemma 1 in the main paper, which states that $\Omega^*(.)$ is continuous and differentiable at R^*. Section 2 proves that a sufficient condition for assumption (a5) in the main paper to hold is that $R(\mathbf{a},\mathbf{q},\mathbf{Q})$ is weakly supermodular in all of its arguments. Section 3 contains the proof of the claim in Section 5 of the main paper that if agents can cooperate in $\mathcal{P}$-mode, they can overcome any attempt by the principal to use team payments to eliminate the double-sided moral hazard problem. Section 4 contains the proof that Lemma 1 still applies in the presence of spillovers. Section 5 provides the derivation of the closed form solutions for the linear model with spillovers used in Section 5.1 of the main paper. Section 6 does likewise for the model with pricing and spillovers used in Section 5.2 of the main paper. Section 7 establishes the result stated at the end of Section 5.1 in the main paper, namely that Proposition 5 continues to hold even if prices are endogenous and contractible, and there are production costs. Finally, Section 8 analyzes the hybrid case in a setting with multiple agents, in which some agents can operate in $\mathcal{P}$-mode and others in $\mathcal{A}$-mode.

1 Proof of Lemma 2

Suppose the optimal allocation of decision rights is $D^* \subset \{1,..,M_a\}$, the proposed optimal contract Ω^* offered by the principal is discontinuous at R^* and $\lim_{R\to R^{*-}} \Omega^*(R) > \lim_{R\to R^{*+}} \Omega^*(R)$. Then

$$\mathbf{Q}^* = \arg\max_{\mathbf{Q}} \{R(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q}) - \Omega^*(R(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q})) - C(\mathbf{Q})\}$$

implies $\Omega^*(R^*) = \lim_{R\to R^{*+}} \Omega^*(R)$, because otherwise $\Omega^*(R^*) > \lim_{R\to R^{*+}} \Omega^*(R)$, so the principal could profitably deviate to, say, $Q^{*1}+\varepsilon$, with ε sufficiently small. But then we must have $\mathbf{q}^* = 0$, since otherwise $q^{*j} > 0$ for some $j \in \{1,..,M_q\}$, so the agent could profitably deviate to $q^{*j}-\varepsilon$ with

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ε sufficiently small. If $\mathbf{q}^* = 0$, then it must be that $\Omega^*(R^*) = 0$ and $D^* = \{1, .., M_a\}$ (i.e. pure $\mathcal{P}$-mode), and therefore

$$(\mathbf{a}^*, \mathbf{Q}^*) = \arg\max_{\mathbf{a},\mathbf{Q}} \{R(\mathbf{a}, \mathbf{0}, \mathbf{Q}) - f(\mathbf{a}) - C(\mathbf{Q})\}. \tag{1}$$

This also means the principal's profits are

$$\Pi^* = \max_{\mathbf{a},\mathbf{Q}} \{R(\mathbf{a}, \mathbf{0}, \mathbf{Q}) - f(\mathbf{a}) - C(\mathbf{Q})\}.$$

In this case the principal could keep $D^* = \{1, .., M_a\}$ but switch to the following linear contract

$$\Omega_\varepsilon(R) = \varepsilon R + c(\mathbf{q}(\varepsilon)) - \varepsilon R(\mathbf{a}(\varepsilon), \mathbf{q}(\varepsilon), \mathbf{Q}(\varepsilon)),$$

where $\varepsilon > 0$ is sufficiently small and $(\mathbf{a}(\varepsilon), \mathbf{q}(\varepsilon), \mathbf{Q}(\varepsilon))$ is a solution to

$$\begin{cases} \mathbf{a}(\varepsilon) = \arg\max_{\mathbf{a}} \{(1-\varepsilon) R(\mathbf{a}, \mathbf{q}(\varepsilon), \mathbf{Q}(\varepsilon)) - f(\mathbf{a})\} \\ \mathbf{q}(\varepsilon) = \arg\max_{\mathbf{q}} \{\varepsilon R(\mathbf{a}(\varepsilon), \mathbf{q}, \mathbf{Q}(\varepsilon)) - c(\mathbf{q})\} \\ \mathbf{Q}(\varepsilon) = \arg\max_{\mathbf{Q}} \{(1-\varepsilon) R(\mathbf{a}(\varepsilon), \mathbf{q}(\varepsilon), \mathbf{Q}) - C(\mathbf{Q})\}. \end{cases}$$

Denote the principal's profit that results from $D^* = \{1, .., M_a\}$ and contract Ω_ε by

$$\Pi^{\mathcal{P}}(\varepsilon) \equiv R(\mathbf{a}(\varepsilon), \mathbf{q}(\varepsilon), \mathbf{Q}(\varepsilon)) - f(\mathbf{a}(\varepsilon)) - c(\mathbf{q}(\varepsilon)) - C(\mathbf{Q}(\varepsilon)).$$

Clearly, $(\mathbf{a}(0), \mathbf{q}(0), \mathbf{Q}(0)) = (\mathbf{a}^*, \mathbf{0}, \mathbf{Q}^*)$ and $\Pi^{\mathcal{P}}(0) = \Pi^*$. We can then use (1), the definition of $\mathbf{q}(\varepsilon)$ and assumption (a2) to obtain

$$\begin{aligned} \Pi_\varepsilon^{\mathcal{P}}(0) &= \sum_{i=1}^{M_a} \left(R_{a^i}(\mathbf{a}^*, \mathbf{0}, \mathbf{Q}^*) - f_{a^i}^i\left(a^{*i}\right)\right) a_\varepsilon^i(0) + \sum_{j=1}^{M_q} R_{q^j}(\mathbf{a}^*, \mathbf{0}, \mathbf{Q}^*)\, q_\varepsilon^j(0) \\ &\quad + \sum_{k=1}^{M_Q} \left(R_{Q^k}(\mathbf{a}^*, \mathbf{0}, \mathbf{Q}^*) - C_{Q^k}^k\left(Q^{*k}\right)\right) Q_\varepsilon^k(0) \\ &= \sum_{j=1}^{M_q} \frac{R_{q^j}(\mathbf{a}^*, \mathbf{0}, \mathbf{Q}^*)^2}{c_{q^j q^j}^j(0)} > 0. \end{aligned}$$

Thus, if $\lim_{R \to R^{*-}} \Omega^*(R) > \lim_{R \to R^{*+}} \Omega^*(R)$, then the principal can keep $D^* = \{1, .., M_a\}$ but profitably deviate to $\Omega_\varepsilon(R)$ for ε small enough, which contradicts the optimality of $\Omega^*(R)$.

The other possibility is $\lim_{R\to R^{*}+}\Omega^{*}(R)>\lim_{R\to R^{*}-}\Omega^{*}(R)$. Then

$$\mathbf{q}^{*}=\arg\max_{\mathbf{q}}\left\{\Omega^{*}\left(R\left(\mathbf{a}^{*},\mathbf{q},\mathbf{Q}^{*}\right)\right)-c\left(\mathbf{q}\right)\right\}$$

implies $\Omega^{*}(R^{*})=\lim_{R\to R^{*}+}\Omega^{*}(R)$, because otherwise $\Omega^{*}(R^{*})<\lim_{R\to R^{*}+}\Omega^{*}(R)$, so the agent could profitably deviate to, say, $q^{*1}+\varepsilon$, with ε sufficiently small. But then we must have $\mathbf{Q}^{*}=0$, otherwise $Q^{*k}>0$ for some $k\in\{1,..,M_{Q}\}$, so the principal could profitably deviate to $Q^{*k}-\varepsilon$ with ε sufficiently small. If $\mathbf{Q}^{*}=0$, then the principal's profits are at most

$$\Pi^{*}\leq R\left(\mathbf{a}^{*},\mathbf{q}^{*},\mathbf{0}\right)-f\left(\mathbf{a}^{*}\right)-c\left(\mathbf{q}^{*}\right)\leq\max_{\mathbf{a},\mathbf{q}}\left\{R\left(\mathbf{a},\mathbf{q},\mathbf{0}\right)-f\left(\mathbf{a}\right)-c\left(\mathbf{q}\right)\right\}.$$

This cannot be optimal. Indeed, the principal could switch to $D=\emptyset$ and the linear contract

$$\widetilde{\Omega}_{\varepsilon}\left(R\right)=\left(1-\varepsilon\right)R+f\left(\widetilde{\mathbf{a}}\left(\varepsilon\right)\right)+c\left(\widetilde{\mathbf{q}}\left(\varepsilon\right)\right)-\left(1-\varepsilon\right)R\left(\widetilde{\mathbf{a}}\left(\varepsilon\right),\widetilde{\mathbf{q}}\left(\varepsilon\right),\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right),$$

where $\varepsilon>0$ is sufficiently small and $\left(\widetilde{\mathbf{a}}\left(\varepsilon\right),\widetilde{\mathbf{q}}\left(\varepsilon\right),\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right)$ is a solution to

$$\begin{cases}\widetilde{\mathbf{a}}\left(\varepsilon\right)=\arg\max_{a}\left\{\left(1-\varepsilon\right)R\left(\mathbf{a},\widetilde{\mathbf{q}}\left(\varepsilon\right),\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right)-f\left(\mathbf{a}\right)\right\}\\ \widetilde{\mathbf{q}}\left(\varepsilon\right)=\arg\max_{q}\left\{\left(1-\varepsilon\right)R\left(\widetilde{\mathbf{a}}\left(\varepsilon\right),\mathbf{q},\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right)-c\left(\mathbf{q}\right)\right\}\\ \widetilde{\mathbf{Q}}\left(\varepsilon\right)=\arg\max_{Q}\left\{\varepsilon R\left(\widetilde{\mathbf{a}}\left(\varepsilon\right),\widetilde{\mathbf{q}}\left(\varepsilon\right),\mathbf{Q}\right)-C\left(\mathbf{Q}\right)\right\}.\end{cases}$$

Denote the principal's profit that results from offering contract $\widetilde{\Omega}_{\varepsilon}$ by

$$\widetilde{\Pi}\left(\varepsilon\right)\equiv R\left(\widetilde{\mathbf{a}}\left(\varepsilon\right),\widetilde{\mathbf{q}}\left(\varepsilon\right),\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right)-f\left(\widetilde{\mathbf{a}}\left(\varepsilon\right)\right)-c\left(\widetilde{\mathbf{q}}\left(\varepsilon\right)\right)-C\left(\widetilde{\mathbf{Q}}\left(\varepsilon\right)\right).$$

Clearly,

$$\begin{aligned}\left(\widetilde{\mathbf{a}}\left(0\right),\widetilde{\mathbf{q}}\left(0\right),\widetilde{\mathbf{Q}}\left(0\right)\right) &= \left(\widetilde{\mathbf{a}}^{*},\widetilde{\mathbf{q}}^{*},\mathbf{0}\right)\equiv\arg\max_{\mathbf{a},\mathbf{q}}\left\{R\left(\mathbf{a},\mathbf{q},\mathbf{0}\right)-f\left(\mathbf{a}\right)-c\left(\mathbf{q}\right)\right\}\\ \widetilde{\Pi}\left(0\right) &= \max_{\mathbf{a},\mathbf{q}}\left\{R\left(\mathbf{a},\mathbf{q},\mathbf{0}\right)-f\left(\mathbf{a}\right)-c\left(\mathbf{q}\right)\right\}\geq\Pi^{*}.\end{aligned}$$

Using the last inequality, the definitions of $\widetilde{\mathbf{a}}(\varepsilon)$ and $\widetilde{\mathbf{Q}}(\varepsilon)$ and assumption (a2), we obtain

$$\begin{aligned}\widetilde{\Pi}_{\varepsilon}(0) &= \sum_{i=1}^{M_a}\left(R_{a^i}\left(\widetilde{\mathbf{a}}^*,\widetilde{\mathbf{q}}^*,\mathbf{0}\right)-f_{a^i}^i\left(\widetilde{a}^{*i}\right)\right)\widetilde{a}_{\varepsilon}^i(0)+\sum_{j=1}^{M_q}\left(R_{q^j}\left(\widetilde{\mathbf{a}}^*,\widetilde{\mathbf{q}}^*,\mathbf{0}\right)-c_{q^j}\left(\widetilde{q}^{*j}\right)\right)\widetilde{q}_{\varepsilon}^j(0)\\ &\quad+\sum_{k=1}^{M_Q}R_{Q^k}\left(\widetilde{\mathbf{a}}^*,\widetilde{\mathbf{q}}^*,\mathbf{0}\right)\widetilde{Q}_{\varepsilon}^k(0)\\ &= \sum_{k=1}^{M_Q}\frac{R_{Q^k}\left(\widetilde{\mathbf{a}}^*,\widetilde{\mathbf{q}}^*,\mathbf{0}\right)^2}{C_{Q^kQ^k}^k(0)}>0.\end{aligned}$$

Thus, the principal can profitably deviate to $D=\emptyset$ and $\widetilde{\Omega}_{\varepsilon}(R)$ for ε small enough, which contradicts the optimality of $\Omega^*(R)$.

We have thus proven that $\lim_{R\to R^{*+}}\Omega^*(R)=\lim_{R\to R^{*-}}\Omega^*(R)$, so Ω^* is continuous at R^*.

Suppose now that Ω^* is non-differentiable at R^* and $\lim_{R\to R^{*+}}\Omega_R^*(R)>\lim_{R\to R^{*-}}\Omega_R^*(R)$. This implies $\mathbf{q}^*=\mathbf{0}$, otherwise there exists $j\in\{1,..,M_q\}$ such that $q^{*j}>0$, so setting q^j slightly below q^{*j} would violate $\mathbf{q}^*=\arg\max_{\mathbf{q}}\{\Omega^*(R(\mathbf{a}^*,\mathbf{q},\mathbf{Q}^*))-c(\mathbf{q})\}$. To see this, let

$$\mathbf{q}_{-j}^*\left(q^j\right)\equiv\left(q^{*1},..,q^{*(j-1)},q^j,q^{*(j+1)},..,q^{*M_q}\right)$$

and note that we must have

$$\begin{aligned}0 &\geq \lim_{q^j\to q^{*j}+}\left\{\Omega_R^*\left(R\left(\mathbf{a}^*,\mathbf{q}_{-j}^*\left(q^j\right),\mathbf{Q}^*\right)\right)R_{q^j}\left(\mathbf{a}^*,\mathbf{q}_{-j}^*\left(q^j\right),\mathbf{Q}^*\right)-c_{q^j}^j\left(q^j\right)\right\}\\ &> \lim_{q^j\to q^{*j}-}\left\{\Omega_R^*\left(R\left(\mathbf{a}^*,\mathbf{q}_{-j}^*\left(q^j\right),\mathbf{Q}^*\right)\right)R_{q^j}\left(\mathbf{a}^*,\mathbf{q}_{-j}^*\left(q^j\right),\mathbf{Q}^*\right)-c_{q^j}^j\left(q^j\right)\right\}.\end{aligned}$$

But $\mathbf{q}^*=\mathbf{0}$ implies that we must have $\Omega^*(R^*)=0$ (recall $c(\mathbf{0})=0$) and $D^*=\{1,..,M_a\}$, so

$$(\mathbf{a}^*,\mathbf{Q}^*)=\arg\max_{\mathbf{a},\mathbf{Q}}\{R(\mathbf{a},\mathbf{0},\mathbf{Q})-f(\mathbf{a})-C(\mathbf{Q})\}.$$

We can then apply the same reasoning as above to conclude that the principal could profitably deviate to the linear contract $\Omega_{\varepsilon}(R)$ for ε small enough.

Suppose instead $\lim_{R\to R^{*+}}\Omega_R^*(R)<\lim_{R\to R^{*-}}\Omega_R^*(R)$. This implies $\mathbf{Q}^*=\mathbf{0}$, otherwise there would exist $k\in\{1,..,M_Q\}$ such that $Q^{*k}>0$, which would then require

$$\begin{aligned}0 &\leq \lim_{Q^k\to Q^{*k}-}\left\{R_{Q^k}\left(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q}_{-k}^*\left(Q^k\right)\right)\left(1-\Omega_R^*\left(R\left(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q}_{-k}^*\left(Q^k\right)\right)\right)\right)-C_{Q^k}^k\left(Q^k\right)\right\}\\ &< \lim_{Q^k\to Q^{*k}+}\left\{R_{Q^k}\left(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q}_{-k}^*\left(Q^k\right)\right)\left(1-\Omega_R^*\left(R\left(\mathbf{a}^*,\mathbf{q}^*,\mathbf{Q}_{-k}^*\left(Q^k\right)\right)\right)\right)-C_{Q^k}^k\left(Q^k\right)\right\}.\end{aligned}$$

Thus, setting Q^k slightly above Q^{*k} would violate

$$\mathbf{Q}^* = \arg\max_{\mathbf{Q}} \left\{ R\left(\mathbf{a}^*, \mathbf{q}^*, \mathbf{Q}\right) - \Omega^*\left(R\left(\mathbf{a}^*, \mathbf{q}^*, \mathbf{Q}\right)\right) - C\left(\mathbf{Q}\right) \right\}.$$

But $\mathbf{Q}^* = \mathbf{0}$ implies that

$$\Pi^* \leq \max_{\mathbf{a},\mathbf{q}} \left\{ R\left(\mathbf{a}, \mathbf{q}, \mathbf{0}\right) - f\left(\mathbf{a}\right) - c\left(\mathbf{q}\right) \right\}.$$

We can then apply the same reasoning as above to conclude that the principal could profitably deviate to $D = \emptyset$ and the linear contract $\widetilde{\Omega}_\varepsilon\left(R\right)$ for ε small enough.

We conclude that $\Omega^*\left(.\right)$ must be continuous and differentiable at R^*.

2 Supermodularity implies (a5)

Recall the definition of $\Pi\left(\boldsymbol{\tau}\right)$ for any vector $\boldsymbol{\tau} \in [0,1]^{M_a+M_q+M_Q}$:

$$\Pi\left(\boldsymbol{\tau}\right) \equiv R\left(\mathbf{a}\left(\boldsymbol{\tau}\right), \mathbf{q}\left(\boldsymbol{\tau}\right), \mathbf{Q}\left(\boldsymbol{\tau}\right)\right) - f\left(\mathbf{a}\left(\boldsymbol{\tau}\right)\right) - c\left(\mathbf{q}\left(\boldsymbol{\tau}\right)\right) - C\left(\mathbf{Q}\left(\boldsymbol{\tau}\right)\right),$$

where $\left(\mathbf{a}\left(\boldsymbol{\tau}\right), \mathbf{q}\left(\boldsymbol{\tau}\right), \mathbf{Q}\left(\boldsymbol{\tau}\right)\right)$ is the unique solution to

$$\begin{cases} \tau^j R_{a^j}\left(\mathbf{a}, \mathbf{q}, \mathbf{Q}\right) = f^j_{a^j}\left(a^j\right) & \text{for } j \in \{1, .., M_a\} \\ \tau^{M_a+k} R_{q^k}\left(\mathbf{a}, \mathbf{q}, \mathbf{Q}\right) = c^k_{q^k}\left(q^k\right) & \text{for } k \in \{1, .., M_q\} \\ \tau^{M_a+M_q+l} R_{Q^l}\left(\mathbf{a}, \mathbf{q}, \mathbf{Q}\right) = C^l_{Q^l}\left(Q^l\right) & \text{for } l \in \{1, .., M_Q\} \end{cases} \tag{2}$$

We wish to prove that if $R\left(\mathbf{a}, \mathbf{q}, \mathbf{Q}\right)$ is (weakly) supermodular in all of its arguments, then $\Pi\left(\boldsymbol{\tau}\right)$ is increasing in τ^i for all $i \in \{1, .., M_a + M_q + M_Q\}$.

We begin by showing that supermodularity implies $\left(\mathbf{a}\left(\boldsymbol{\tau}\right), \mathbf{q}\left(\boldsymbol{\tau}\right), \mathbf{Q}\left(\boldsymbol{\tau}\right)\right)$ is increasing in τ^i. To do so, note that the solution $\left(\mathbf{a}\left(\boldsymbol{\tau}\right), \mathbf{q}\left(\boldsymbol{\tau}\right), \mathbf{Q}\left(\boldsymbol{\tau}\right)\right)$ to (2) corresponds to a game in which there are $M_a + M_q + M_Q$ players, and where each player $j \in \{1, .., M_a\}$ sets

$$a^j = \arg\max_a \left\{ \tau^j R\left(a^1, .., a^{j-1}, a, a^{j+1}, .., a^{M_a}, \mathbf{q}, \mathbf{Q}\right) - f^j\left(a\right) \right\};$$

each player $M_a + k$ for $k \in \{1, .., M_q\}$ sets

$$q^k = \arg\max_q \left\{ \tau^{M_a+k} R\left(\mathbf{a}, q^1, .., q^{k-1}, q, q^{k+1}, .., q^{M_q}, \mathbf{Q}\right) - c^k\left(q\right) \right\};$$

and each player $M_a + M_q + l$ for $l \in \{1, .., M_Q\}$ sets

$$Q^l = \arg\max_Q \left\{ \tau^{M_a+M_q+l} R\left(\mathbf{a}, \mathbf{q}, Q^1, .., Q^{l-1}, Q, Q^{l+1}, .., Q^{M_Q}\right) - C^l(Q) \right\}.$$

Since $R(\mathbf{a}, \mathbf{q}, \mathbf{Q})$ is supermodular in all of its arguments, this game is supermodular, with payoffs having weakly increasing differences in the actions and the parameters $\left(\tau^1, .., \tau^{M_a+M_q+M_Q}\right)$. From standard supermodularity results (Vives, 1999), we know that an increase in any of the parameters $\left(\tau^1, .., \tau^{M_a+M_q+M_Q}\right)$ will increase each of the solutions $a^j(\boldsymbol{\tau})$ for $j \in \{1, .., M_a\}$, $q^k(\boldsymbol{\tau})$ for $k \in \{1, .., M_q\}$ and $Q^l(\boldsymbol{\tau})$ for $l \in \{1, .., M_Q\}$ in a weak sense. To obtain the strict increase, note that if τ^i increases for some i and no $a^j(\boldsymbol{\tau})$, $q^k(\boldsymbol{\tau})$ or $Q^l(\boldsymbol{\tau})$ increases, then no $a^j(\boldsymbol{\tau})$, $q^k(\boldsymbol{\tau})$ or $Q^l(\boldsymbol{\tau})$ can change since none can decrease. But if $(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau}))$ remain unchanged, then, since τ^i is higher, the first-order conditions (2) can no longer hold. Thus, at least one $a^j(\boldsymbol{\tau})$ or one $q^k(\boldsymbol{\tau})$ or one $Q^l(\boldsymbol{\tau})$ must increase.

Next, for any $i \in \{1, .., M_a + M_q + M_Q\}$, we have

$$\begin{aligned}
\frac{d\Pi(\boldsymbol{\tau})}{d\tau^i} &= \sum_{j=1}^{M_a} \left(R_{a^j}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) - f^j_{a^j}\left(a^j(\boldsymbol{\tau})\right) \right) \frac{da^j}{d\tau^i} + \sum_{k=1}^{M_q} \left(R_{q^k}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) - c^k_{q^k}\left(q^k(\boldsymbol{\tau})\right) \right) \frac{dq^k}{d\tau^i} \\
&\quad + \sum_{l=1}^{M_Q} \left(R_{Q^l}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) - C^l_{Q^l}\left(Q^l(\boldsymbol{\tau})\right) \right) \frac{dQ^l}{d\tau^i} \\
&= \sum_{j=1}^{M_a} \left(1 - \tau^j\right) R_{a^j}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) \frac{da^j}{d\tau^i} + \sum_{k=1}^{M_q} \left(1 - \tau^{M_a+k}\right) R_{q^k}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) \frac{dq^k}{d\tau^i} \\
&\quad + \sum_{l=1}^{M_Q} \left(1 - \tau^{M_a+M_q+l}\right) R_{Q^l}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) \frac{dQ^l}{d\tau^i},
\end{aligned}$$

where we have used (2) to replace $f^j_{a^j}\left(a^j(\boldsymbol{\tau})\right)$, $c^k_{q^k}\left(q^k(\boldsymbol{\tau})\right)$ and $C^l_{Q^l}\left(Q^l(\boldsymbol{\tau})\right)$. Assumption (a2) implies $R_{a^j} > 0$, $R_{q^k} > 0$ and $R_{Q^l} > 0$ for all $j \in \{1, .., M_a\}$, $k \in \{1, .., M_q\}$ and $l \in \{1, .., M_Q\}$. Furthermore, since $(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau}))$ is increasing in τ^i, we have $\frac{da^j(\boldsymbol{\tau})}{d\tau^i} \geq 0$, $\frac{dq^k(\boldsymbol{\tau})}{d\tau^i} \geq 0$ and $\frac{dQ^l(\boldsymbol{\tau})}{d\tau^l} \geq 0$ for all $j \in \{1, .., M_a\}$, $k \in \{1, .., M_q\}$ and $l \in \{1, .., M_Q\}$, with at least one strict inequality—suppose the strict inequality occurs for a^j. This implies $\left(1 - \tau^j\right) R_{a^j}(\mathbf{a}(\boldsymbol{\tau}), \mathbf{q}(\boldsymbol{\tau}), \mathbf{Q}(\boldsymbol{\tau})) \frac{da^j}{d\tau^i} > 0$, therefore we can conclude that $\frac{d\Pi(\boldsymbol{\tau})}{d\tau^i} > 0$ for any $\boldsymbol{\tau} \in [0, 1)^{M_a+M_q+M_Q}$.

3 Ruling out team payments

We consider the setting in Section 5 of the main paper in which there are multiple agents and spillovers, and the principal attempts to use team payments in $\mathcal{P}$-mode. Suppose the principal offers the contract

(t, τ, T) to each agent i, which implies the net payoff to agent i is

$$(1-t) R_i - \tau \overline{R}_{-i} - c(q_i) - T,$$

where R_i indicates the revenue attributable to agent i and $\overline{R}_{-i}$ indicates the average revenue across all other agents. Because of spillovers, each agent's revenue can depend on all other agents' transferable actions. Note τ can be positive or negative, and reflects the possibility of team payments. By setting $t = 0$ and $\tau = 1$, each agent's moral hazard problem is entirely solved, while the principal still collects all the variable revenue and fully internalizes the spillovers across transferable actions.

However, any such scheme can be undone if the agents can cooperate. Indeed, cooperation means that agents will jointly choose $(q_1, .., q_N)$ to maximize

$$(1-t-\tau) \sum_{i=1}^{N} R_i - \sum_{i=1}^{N} c(q_i).$$

Thus, everything is as if the principal offers agents a standard linear revenue-sharing contract $\left(\widetilde{t}, T\right)$ with no team payments, where

$$\widetilde{t} = t + \tau.$$

In particular, if $t = 0$ and $\tau = 1$, then each agent i chooses $q_i = 0$, the least efficient outcome from an agent moral hazard perspective. Consequently, allowing for team payments cannot improve upon the outcome in $\mathcal{P}$-mode with team payments ruled out.[1]

4 Proof of modified Lemma 1 for the case with spillovers

(a1') All functions are twice continuously differentiable in their arguments.

(a2') The cost functions c and C are increasing and convex in their arguments. If $f \neq 0$, then f is also increasing and convex. The revenue function $R(a_i, \sigma_i, q_i, Q)$ is non-negative and increasing in (q_i, Q). If $f \neq 0$, then $R(a_i, \sigma_i, q_i, Q)$ is increasing in a_i and $\sum_{i=1}^{N} R(a_i, \sigma(a_{-i}), q_i, Q)$ is increasing in each a_i, for $i \in \{1, .., N\}$. Furthermore,

$$f(0) = f_a(0) = c(0) = c_q(0) = C(0) = C_Q(0) = 0.$$

(a3') For all $(\sigma_i, Q) \in \mathbb{R}_+^2$ and $t \in (0, 1]$, $tR(a_i, \sigma_i, q_i, Q) - f(a_i) - c(q_i)$ is concave and admits a

[1] Note that this conclusion holds when there are no spillovers across the q_i's (the principal already internalizes the spillovers across the a_i's in $\mathcal{P}$-mode).

unique finite maximizer in (a_i, q_i). *For all* $t \in (0,1]$ *and* $(q_1, .., q_N) \in \mathbb{R}_+^N$,

$$\sum_{i=1}^{N} (tR(a_i, \sigma(a_{-i}), q_i, Q) - f(a_i) - c(q_i)) - C(Q)$$

is concave and admits a unique finite maximizer in $(a_1, .., a_N, Q)$, which is symmetric in $(a_1, .., a_N)$.
(a4') For any $(t_1, t_2, t_3) \in [0,1]^3$ *and* $I \in \{0,1\}$, *the following system of equations admits a unique solution* (a, q, Q)*:*

$$\begin{cases} t_1 \left(R_a(a, \sigma(a, .., a), q, Q) + I \frac{d(\sigma(a,..,a))}{da} R_\sigma(a, \sigma(a, .., a), q, Q) \right) = f_a(a) \\ t_2 R_q(a, \sigma(a, .., a), q, Q) = c_q(q) \\ t_3 N R_Q(a, \sigma(a, .., a), q, Q) = C_Q(Q). \end{cases}$$

We now prove that under assumptions (a1')-(a4'), Lemma 1 from the main paper holds for the case with spillovers, i.e. the principal can achieve the best possible outcome with a linear contract in both modes.

Consider first the $\mathcal{P}$-mode. Denote by

$$[a]_n \equiv \underbrace{(a, ..., a)}_{n},$$

the vector of $n \in \{1, .., N\}$ coordinates all equal to a. Since we have assumed that it is optimal for the principal to induce all N agents to join and we are focusing on symmetric solutions, the optimal contract $\Omega^*(.)$ (i.e. payment to each agent) solves

$$\begin{aligned} \Pi^{\mathcal{P}*} &= \max_{\Omega(.),Q,a,q} \left\{ N \left[R\left(a, \sigma\left([a]_{N-1}\right), q, Q\right) - \Omega\left(R\left(a, \sigma\left([a]_{N-1}\right), q, Q\right)\right) - f(a) \right] - C(Q) \right\} \quad (3) \\ & \text{s.t.} \\ & a = \arg\max_{a'} \left\{ \begin{array}{c} R\left(a', \sigma\left([a]_{N-1}\right), q, Q\right) - \Omega\left(R\left(a', \sigma\left([a]_{N-1}\right), q, Q\right)\right) \\ + (N-1)\left(R\left(a, \sigma\left(a', [a]_{N-2}\right), q, Q\right) - \Omega\left(R\left(a, \sigma\left(a', [a]_{N-2}\right), q, Q\right)\right)\right) - f(a') \end{array} \right\} \\ & q = \arg\max_{q'} \left\{ \Omega\left(R\left(a, \sigma\left([a]_{N-1}\right), q', Q\right)\right) - c(q') \right\} \\ & Q = \arg\max_{Q'} \left\{ N \left[R\left(a, \sigma\left([a]_{N-1}\right), q, Q'\right) - \Omega\left(R\left(a, \sigma\left([a]_{N-1}\right), q, Q'\right)\right) \right] - C(Q') \right\} \\ & 0 \le \Omega\left(R\left(a, \sigma\left([a]_{N-1}\right), q, Q\right)\right) - c(q). \end{aligned}$$

Let then (a^*, q^*, Q^*) denote the symmetric outcome of this optimization problem. Also define

$$R^* \equiv R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q^*\right).$$

Assume for the moment that $\Omega^*(.)$ is continuous and differentiable at R^*.

The program (3) implies that (a^*, q^*, Q^*) solve

$$\begin{cases} \left(1-\Omega_R^*(R^*)\right)\left(R_a\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q^*\right)+\frac{d\left(\sigma\left([a]_{N-1}\right)\right)}{da}_{a=a^*}R_\sigma\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q^*\right)\right)=f_a(a^*) \\ \Omega_R^*(R^*)\,R_q\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q^*\right)=c_q(q^*) \\ N\left(1-\Omega_R^*(R^*)\right)R_Q\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q^*\right)=C_Q(Q^*)\,. \end{cases}$$

Let then $t^* \equiv 1-\Omega_R^*(R^*)$ and $T^* \equiv (1-t^*)R^* - \Omega^*(R^*)$. Clearly, the linear contract $\widehat{\Omega}(R) = (1-t^*)R - T^*$ can generate the same stage-2 symmetric Nash equilibrium (a^*, q^*, Q^*) as the initial contract $\Omega^*(R)$. Furthermore, both $\Omega^*(R)$ and $\widehat{\Omega}(R)$ cause the agents' participation constraint to bind and therefore result in the same profits for the principal.

Now it remains to show that $\Omega^*(.)$ is continuous and differentiable at R^*.

Suppose the contract Ω^* offered by the principal is discontinuous at R^* and $\lim_{R\to R^{*-}}\Omega^*(R) > \lim_{R\to R^{*+}}\Omega^*(R)$. Then

$$Q^* = \arg\max_Q \left\{N\left(R\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q\right) - \Omega^*\left(R\left(a^*,\sigma\left([a^*]_{N-1}\right),q^*,Q\right)\right)\right) - C(Q)\right\}$$

implies $\Omega^*(R^*) = \lim_{R\to R^{*+}}\Omega^*(R)$, because otherwise $\Omega^*(R^*) > \lim_{R\to R^{*+}}\Omega^*(R)$, so the principal could profitably deviate to $Q^*+\varepsilon$, with ε sufficiently small. But then we must have $q^*=0$, since otherwise $q^*>0$ and any agent could profitably deviate to $q^*-\varepsilon$ with ε sufficiently small. If $q^*=0$, then it must be that $\Omega^*(R^*)=0$ and therefore

$$(a^*, Q^*) = \arg\max_{a,Q}\left\{N\left(R\left(a,\sigma\left([a]_{N-1}\right),0,Q\right) - f(a)\right) - C(Q)\right\}. \tag{4}$$

In this case the principal could switch to the following linear contract

$$\Omega_\varepsilon(R) = \varepsilon R + c(q(\varepsilon)) - \varepsilon R\left(a(\varepsilon),\sigma\left([a(\varepsilon)]_{N-1}\right),q(\varepsilon),Q(\varepsilon)\right),$$

where $\varepsilon>0$ is sufficiently small and $(a(\varepsilon), q(\varepsilon), Q(\varepsilon))$ is a solution to

$$\begin{cases} a(\varepsilon) = \arg\max_a\left\{(1-\varepsilon)R\left(a,\sigma\left([a]_{N-1}\right),q(\varepsilon),Q(\varepsilon)\right) - f(a)\right\} \\ q(\varepsilon) = \arg\max_q\left\{\varepsilon R\left(a(\varepsilon),\sigma\left([a(\varepsilon)]_{N-1}\right),q,Q(\varepsilon)\right) - c(q)\right\} \\ Q(\varepsilon) = \arg\max_Q\left\{N(1-\varepsilon)R\left(a(\varepsilon),\sigma\left([a(\varepsilon)]_{N-1}\right),q(\varepsilon),Q\right) - C(Q)\right\}. \end{cases}$$

Denote the principal's profit that results from offering contract Ω_ε by

$$\Pi^{\mathcal{P}}(\varepsilon) \equiv N\left(R\left(a(\varepsilon), \sigma\left([a(\varepsilon)]_{N-1}\right), q(\varepsilon), Q(\varepsilon)\right) - f(a(\varepsilon)) - c(q(\varepsilon))\right) - C(Q(\varepsilon)).$$

Clearly, $(a(0), q(0), Q(0)) = (a^*, 0, Q^*)$ and $\Pi^{\mathcal{P}}(0) = \Pi^{\mathcal{P}*}$. We can then use (4), the definition of $q(\varepsilon)$ and assumption (a2') to obtain

$$\begin{aligned}
\Pi^{\mathcal{P}}_\varepsilon(0) &= N\left(R_a\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right) + \left.\frac{d\left(\sigma\left([a]_{N-1}\right)\right)}{da}\right|_{a=a^*} R_\sigma\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right) - f_a(a^*)\right) a_\varepsilon(0) \\
&\quad + N\left(R_q\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right) - c_q(0)\right) q_\varepsilon(0) + \left(NR_Q\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right) - C_Q(Q^*)\right) Q_\varepsilon(0) \\
&= NR_q\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right) \frac{R_q\left(a^*, \sigma\left([a^*]_{N-1}\right), 0, Q^*\right)}{c_{qq}(0)} > 0,
\end{aligned}$$

where $c_{qq} = \frac{d^2c}{dq^2}$.

Thus, if $\lim_{R\to R^{*-}} \Omega^*(R) > \lim_{R\to R^{*+}} \Omega^*(R)$, then the principal can profitably deviate to $\Omega_\varepsilon(R)$ for ε small enough, which contradicts the optimality of $\Omega^*(R)$.

The other possibility is $\lim_{R\to R^{*+}} \Omega^*(R) > \lim_{R\to R^{*-}} \Omega^*(R)$. Then

$$q^* = \arg\max_q \left\{\Omega^*\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right)\right) - c(q)\right\}$$

implies $\Omega^*(R^*) = \lim_{R\to R^{*+}} \Omega^*(R)$, because otherwise $\Omega^*(R^*) < \lim_{R\to R^{*+}} \Omega^*(R)$, so any agent could profitably deviate to $q^* + \varepsilon$, with ε sufficiently small. But then we must have $Q^* = 0$, otherwise the principal could profitably deviate to $Q^* - \varepsilon$ with ε sufficiently small.

If a is not price, then $NR(a, s(a), q^*, Q^*)$ is increasing in a by assumption (a2'), so the same logic implies $a^* = 0$. We must then have

$$\Pi^{\mathcal{P}*} = N\left(R\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right) - c(q^*)\right) = N \max_q \left\{R\left(0, \sigma\left([0]_{N-1}\right), q, 0\right) - c(q)\right\}. \quad (5)$$

This cannot be optimal. Indeed, the principal could switch to the linear contract

$$\widetilde{\Omega}_\varepsilon(R) = (1-\varepsilon) R + c(\widetilde{q}(\varepsilon)) - (1-\varepsilon) R\left(\widetilde{a}(\varepsilon), \sigma\left([\widetilde{a}(\varepsilon)]_{N-1}\right), \widetilde{q}(\varepsilon), \widetilde{Q}(\varepsilon)\right),$$

where $\varepsilon > 0$ is sufficiently small and $\left(\widetilde{a}(\varepsilon), \widetilde{q}(\varepsilon), \widetilde{Q}(\varepsilon)\right)$ is a solution to

$$\begin{cases} \widetilde{a}(\varepsilon) = \arg\max_a \left\{\varepsilon R\left(a, \sigma\left([a]_{N-1}\right), \widetilde{q}(\varepsilon), \widetilde{Q}(\varepsilon)\right) - f(a)\right\} \\ \widetilde{q}(\varepsilon) = \arg\max_q \left\{(1-\varepsilon) R\left(\widetilde{a}(\varepsilon), \sigma\left([\widetilde{a}(\varepsilon)]_{N-1}\right), q, \widetilde{Q}(\varepsilon)\right) - c(q)\right\} \\ \widetilde{Q}(\varepsilon) = \arg\max_Q \left\{N\varepsilon R\left(\widetilde{a}(\varepsilon), \sigma\left([\widetilde{a}(\varepsilon)]_{N-1}\right), \widetilde{q}(\varepsilon), Q\right) - C(Q)\right\}. \end{cases}$$

Denote the principal's profit that results from offering contract $\widetilde{\Omega}_\varepsilon$ by

$$\widetilde{\Pi}^{\mathcal{P}}(\varepsilon) \equiv N\left(R\left(\widetilde{a}(\varepsilon), \sigma\left([\widetilde{a}(\varepsilon)]_{N-1}\right), \widetilde{q}(\varepsilon), \widetilde{Q}(\varepsilon)\right) - f(\widetilde{a}(\varepsilon)) - c(\widetilde{q}(\varepsilon))\right) - C\left(\widetilde{Q}(\varepsilon)\right).$$

Clearly, $\left(\widetilde{a}(0), \widetilde{q}(0), \widetilde{Q}(0)\right) = (0, q^*, 0)$ and $\widetilde{\Pi}^{\mathcal{P}}(0) = \Pi^{\mathcal{P}*}$. Using (5), the definitions of $\widetilde{a}(\varepsilon)$ and $\widetilde{Q}(\varepsilon)$ and assumption (a2'), we obtain

$$\begin{aligned} \widetilde{\Pi}^{\mathcal{P}}_\varepsilon(0) &= N\left(R_a\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right) + \frac{d\left(\sigma\left([a]_{N-1}\right)\right)}{da}\bigg|_{a=0} R_\sigma\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right)\right)\widetilde{a}_\varepsilon(0) \\ &\quad + N\left(R_q\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right) - c_q(q^*)\right)\widetilde{q}_\varepsilon(0) + N R_Q\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right)\widetilde{Q}_\varepsilon(0) \\ &= N\frac{\left(R_a\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right) + s_a(0) R_\sigma\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right)\right)^2}{f_{aa}(0)} + N\frac{\left(R_Q\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right)\right)^2}{C_{QQ}(0)} > 0, \end{aligned}$$

where $f_{aa} = \frac{d^2 f}{da^2}$ and $C_{QQ} = \frac{d^2 C}{dQ^2}$.

Thus, the principal can profitably deviate to $\widetilde{\Omega}_\varepsilon(R)$ for ε small enough, which contradicts the optimality of $\Omega^*(R)$.

If a is price, then $f = 0$ so we must have

$$\Pi^{\mathcal{P}*} = R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, 0\right) - c(q^*) \leq \max_{a,q}\left\{R\left(a, \sigma\left([a]_{N-1}\right), q, 0\right) - c(q)\right\}.$$

Once again, it is straightforward to verify that the principal could profitably deviate to $\widetilde{\Omega}_\varepsilon(R)$ for ε small enough.

We have thus proven that $\lim_{R\to R^{*+}} \Omega^*(R) = \lim_{R\to R^{*-}} \Omega^*(R)$, so Ω^* is continuous at R^*.

Suppose now that Ω^* is non-differentiable at R^* and $\lim_{R\to R^{*+}} \Omega^*_R(R) > \lim_{R\to R^{*-}} \Omega^*_R(R)$. This implies $q^* = 0$, otherwise we must have

$$\begin{aligned} 0 &\geq \lim_{q\to q^{*+}}\left\{\Omega^*_R\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right)\right) R_q\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right) - c_q(q)\right\} \\ &> \lim_{q\to q^{*-}}\left\{\Omega^*_R\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right)\right) R_q\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right) - c_q(q)\right\}, \end{aligned}$$

so setting q slightly below q^* would violate $q^* = \arg\max_q \left\{\Omega\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q, Q^*\right)\right) - c(q)\right\}$. If $q^* = 0$, then we must have $\Omega^*(R^*) = 0$ (recall $c(0) = 0$) and therefore

$$(a^*, Q^*) = \arg\max_{a,Q} \left\{N\left(R\left(a, \sigma\left([a]_{N-1}\right), 0, Q\right) - f(a)\right) - C(Q)\right\}.$$

But then we can apply the same reasoning as above to conclude that the principal could profitably deviate to the linear contract $\Omega_\varepsilon(R)$ for ε small enough.

Suppose instead $\lim_{R\to R^{*+}} \Omega_R^*(R) < \lim_{R\to R^{*-}} \Omega_R^*(R)$. This implies $Q^* = 0$, otherwise we must have

$$\begin{aligned} 0 &\leq \lim_{Q\to Q^{*-}} \left\{NR_Q\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right)\left(1 - \Omega_R^*\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right)\right)\right) - C_Q(Q)\right\} \\ &< \lim_{Q\to Q^{*+}} \left\{NR_Q\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right)\left(1 - \Omega_R^*\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right)\right)\right) - C_Q(Q)\right\}, \end{aligned}$$

so setting Q slightly above Q^* would violate

$$Q^* = \arg\max_{Q} \left\{N\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right) - \Omega\left(R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, Q\right)\right)\right) - C(Q)\right\}.$$

If action a is not price, then $f \neq 0$ and $NR\left(a, \sigma\left([a]_{N-1}\right), q^*, Q^*\right)$ is increasing in a by assumption (a3), so that the exact same reasoning applies to a^* and leads to $a^* = 0$. This would mean that

$$\Pi^{\mathcal{P}*} = R\left(0, \sigma\left([0]_{N-1}\right), q^*, 0\right) - c(q^*) = \max_{q} \left\{R\left(0, \sigma\left([0]_{N-1}\right), q, 0\right) - c(q)\right\}.$$

We have already proven above that this cannot be optimal.

If action a is price, then $f = 0$ and

$$\Pi^{\mathcal{P}*} = R\left(a^*, \sigma\left([a^*]_{N-1}\right), q^*, 0\right) - c(q^*) \leq \max_{a,q} \left\{R\left(a, \sigma\left([a]_{N-1}\right), q, 0\right) - c(q)\right\}.$$

In this case, we have proven above that the principal could do better with the linear contract $\widetilde{\Omega}_\varepsilon(R)$ for ε small enough.

We conclude that $\Omega^*(.)$ must be continuous and differentiable at R^*. A similar proof applies to the case when the principal chooses the $\mathcal{A}$-mode.

5 Linear example

Consider first the $\mathcal{P}$-mode. The payoff to agent i from working for the principal is

$$(1-t)\,R_i - \frac{1}{2}q_i^2 - T = (1-t)\left(\beta a_i + x\left(\overline{a}_{-i} - a_i\right) + \phi q_i + \Phi Q\right) - \frac{\phi}{2}q_i^2 - T,$$

which implies that the level of effort chosen by each agent in the second stage is

$$q^{\mathcal{P}}\left(t\right) = 1 - t.$$

In $\mathcal{P}$-mode, the principal sets $a_1, ..., a_N$ and Q to maximize its second stage revenues (wages are paid in the first stage):

$$\sum_{i=1}^{N}\left(t\left(\beta a_i + x\left(\overline{a}_{-i} - a_i\right) + \phi q_i + \Phi Q\right) - \frac{\beta}{2}a_i^2\right) - \frac{\Phi}{2}Q^2,$$

implying the principal's optimal choices are

$$\begin{aligned} a^{\mathcal{P}}\left(t\right) &= t \\ Q^{\mathcal{P}}\left(t\right) &= Nt. \end{aligned}$$

The fixed fee T is set to render each agent indifferent between working for the principal and her outside option, so the expression of $\mathcal{P}$-mode profits as a function of t is

$$\frac{N}{2}\left(\left(\beta + N\Phi\right)t\left(2-t\right) + \phi\left(1 - t^2\right)\right). \tag{6}$$

Maximizing (6) with respect to t implies the optimal variable fee in $\mathcal{P}$-mode is

$$t^{\mathcal{P}*} = \frac{\beta + N\Phi}{\beta + \phi + N\Phi},$$

which is positive but smaller than 1. With this optimal fee, the resulting profits in $\mathcal{P}$-mode are

$$\Pi^{\mathcal{P}*} = \frac{N}{2}\left(\beta + N\Phi + \frac{\phi^2}{\beta + \phi + N\Phi}\right). \tag{7}$$

Consider next the $\mathcal{A}$-mode. The payoff to an individual agent joining the principal is

$$(1-t)\left(\beta a_i + x\left(\overline{a}_{-i} - a_i\right) + \phi q_i + \Phi Q\right) - \frac{\beta}{2}a_i^2 - \frac{\phi}{2}q_i^2 - T.$$

Individual agents maximize their second stage payoff by choosing

$$q^{\mathcal{A}}(t) = 1 - t$$
$$a^{\mathcal{A}}(t) = \left(1 - \tfrac{x}{\beta}\right)(1 - t).$$

The principal's second stage profits in $\mathcal{A}$-mode are

$$\sum_{i=1}^{N} t\left(\beta a_i + x\left(\overline{a}_{-i} - a_i\right) + \phi q_i + \Phi Q\right) - \frac{\Phi}{2} Q^2,$$

which the principal maximizes over Q, leading to

$$Q^{\mathcal{A}}(t) = Nt.$$

Stepping back to the first stage, the principal sets T to equalize the agents' net payoff to their outside option. Total profit for the principal in $\mathcal{A}$-mode as a function of t is then

$$\frac{N}{2}\left(\left(1 - \frac{x}{\beta}\right)(1 - t)\left(\beta + x + (\beta - x)t\right) + \phi\left(1 - t^2\right) + N\Phi t(2 - t)\right). \tag{8}$$

The optimal variable fee is

$$t^{\mathcal{A}*} = \frac{N\Phi - \frac{x}{\beta}(\beta - x)}{\frac{1}{\beta}(\beta - x)^2 + \phi + N\Phi}.$$

Resulting profits in $\mathcal{A}$-mode are

$$\Pi^{\mathcal{A}*} = \frac{N}{2}\left(\beta - \frac{x^2}{\beta} + \phi + \frac{\left(N\Phi - \frac{x}{\beta}(\beta - x)\right)^2}{\frac{1}{\beta}(\beta - x)^2 + \phi + N\Phi}\right). \tag{9}$$

Comparing (7) with (9), the $\mathcal{A}$-mode is preferred if and only if

$$\phi + \frac{\left(N\Phi - \frac{x}{\beta}(\beta - x)\right)^2}{\frac{1}{\beta}(\beta - x)^2 + \phi + N\Phi} > N\Phi + \frac{x^2}{\beta} + \frac{\phi^2}{\beta + \phi + N\Phi}.$$

If there are no spillovers, i.e. $x = 0$, then this condition simplifies to

$$\phi > N\Phi.$$

For $x \neq 0$, the condition can be re-written

$$\left| \frac{\phi x}{\beta} + \beta + N\Phi \right| \leq \sqrt{\beta \left(\beta + \phi + N\Phi \right) + \phi^2}.$$

Finally, let us determine the effects of ϕ and Φ on the tradeoff between the two modes. To do so, we apply the envelope theorem to expressions (6) and (8) and obtain

$$\begin{aligned} \frac{d\Pi^{\mathcal{P}*}}{d\phi} &= \frac{N}{2}\left(1 - \left(t^{\mathcal{P}*}\right)^2\right) \text{ and } \frac{d\Pi^{\mathcal{P}*}}{d\left(N\Phi^2\right)} = \frac{N}{2} t^{\mathcal{P}*}\left(2 - t^{\mathcal{P}*}\right) \\ \frac{d\Pi^{\mathcal{A}*}}{d\phi} &= \frac{N}{2}\left(1 - \left(t^{\mathcal{A}*}\right)^2\right) \text{ and } \frac{d\Pi^{\mathcal{A}*}}{d\left(N\Phi^2\right)} = \frac{N}{2} t^{\mathcal{A}*}\left(2 - t^{\mathcal{A}*}\right). \end{aligned}$$

Since $0 < t^{\mathcal{P}*}, t^{\mathcal{A}*} < 1$ and $t\left(2-t\right)$ is increasing in t for $t \in [0,1]$, we conclude that

$$\begin{aligned} \frac{d\left(\Pi^{\mathcal{A}*} - \Pi^{\mathcal{P}*}\right)}{d\left(\phi^2\right)} &> 0 \text{ if and only if } t^{\mathcal{P}*} > t^{\mathcal{A}*} \\ \frac{d\left(\Pi^{\mathcal{P}*} - \Pi^{\mathcal{A}*}\right)}{d\left(N\Phi^2\right)} &> 0 \text{ if and only if } t^{\mathcal{P}*} > t^{\mathcal{A}*}. \end{aligned}$$

6 Linear example: endogenous price and production costs

We now extend the linear example by allowing the principal to also set a price in the contracting stage, along with the fees (t, T), and by also adding a production cost. We will establish the result stated at the end of Section 5.1 in the main paper, i.e. that Proposition 5 continues to hold in this case.

The revenue generated by agent i is now

$$R\left(p, a_i, q_i, Q\right) = \left(p - d\right)\left(D_0 + \beta a_i + x\left(\overline{a}_{-i} - a_i\right) + \phi q_i + \Phi Q - p\right),$$

where $d \geq 0$ is a constant marginal production cost, p is the price chosen by the principal and D_0 is some baseline level of demand. Fixed costs are still quadratic

$$f\left(a\right) = \frac{\beta}{2}a^2, \quad c\left(q\right) = \frac{\phi}{2}q^2 \quad \text{and} \quad C\left(Q\right) = \frac{\Phi}{2}Q^2.$$

First, we show that whether the production cost is incurred by the principal or the agent does not affect profits in either mode. In $\mathcal{P}$-mode, if the principal incurs the production cost, then the maximization problem is[2]

$$\widetilde{\Pi}^{\mathcal{P}*} = \max_{p,t,a,q,Q} \left\{ N\left((p-d)\left(D_0 + \beta a + \phi q + \Phi Q - p\right) - \frac{\beta}{2}a^2 - \frac{\phi}{2}q^2 \right) - \frac{\Phi}{2}Q^2 \right\}$$

$$\text{s.t.} \quad \begin{cases} (tp-d) = a \\ (1-t)\,p = q \\ (tp-d)\,N = Q. \end{cases}$$

If instead the agent incurs the production cost, then the maximization problem is

$$\widetilde{\Pi}^{\mathcal{P}*} = \max_{p,\widetilde{t},a,q,Q} \left\{ N\left((p-d)\left(D_0 + \beta a + \phi q + \Phi Q - p\right) - \frac{\beta}{2}a^2 - \frac{\phi}{2}q^2 \right) - \frac{\Phi}{2}Q^2 \right\}$$

$$\text{s.t.} \quad \begin{cases} \widetilde{t}p = a \\ \left(\left(1-\widetilde{t}\right)p - d\right) = q \\ \widetilde{t}pN = Q. \end{cases}$$

By making the change of variables $\widetilde{t} \equiv t - \frac{d}{p}$, the second maximization problem becomes the same as the first.

Similarly, in $\mathcal{A}$-mode, if the principal incurs the production cost, then the maximization problem is

$$\widetilde{\Pi}^{\mathcal{A}*} = \max_{p,t,a,q,Q} \left\{ N\left((p-d)\left(D_0 + \beta a + \phi q + \Phi Q - p\right) - \frac{\beta}{2}a^2 - \frac{\phi}{2}q^2 \right) - \frac{\Phi}{2}Q^2 \right\}$$

$$\text{s.t.} \quad \begin{cases} (1-t)\,p\left(1-\frac{x}{\beta}\right) = a \\ (1-t)\,p = q \\ (tp-d)\,N = Q. \end{cases}$$

[2]Given symmetry across the N agents and the formulation of spillovers in this example, they have no impact on the optimization problem in $\mathcal{P}$-mode and only affect the first-order condition in a in $\mathcal{A}$-mode. Furthermore, the analysis that follows would be identical if we allowed for spillovers across the choices of prices. These spillovers would have no impact on the resulting tradeoff because they are internalized in both modes by the principal when it sets prices in the contracting stage.

If instead the agent incurs the production cost, then the maximization problem is

$$
\begin{aligned}
\widetilde{\Pi}^{\mathcal{A}*} &= \max_{p,\widetilde{t},a,q,Q} \left\{ N\left((p-d)(D_0+\beta a+\phi q+\Phi Q-p) - \frac{\beta}{2}a^2 - \frac{\phi}{2}q^2 \right) - \frac{\Phi}{2}Q^2 \right\} \\
& \text{s.t.} \\
& \left\{ \begin{array}{c} \left(\left(1-\widetilde{t}\right)p-d\right)\left(1-\frac{x}{\beta}\right)=a \\ \left(\left(1-\widetilde{t}\right)p-d\right)=q \\ \widetilde{t}pN=Q. \end{array} \right.
\end{aligned}
$$

Again, by making the change of variables $\widetilde{t}=t-\frac{d}{p}$, the second maximization problem becomes the same as the first. Thus, in our setting it is irrelevant which party actually incurs the production cost.

Solving the program above in $\mathcal{P}$-mode, we obtain

$$
\widetilde{\Pi}^{\mathcal{P}*} = N\max_{p,t}\left\{ (p-d)(D_0-p) + \frac{\beta+N\Phi}{2}(tp-d)((2-t)p-d) + \frac{\phi}{2}p(1-t)(p(1+t)-2d) \right\}.
$$

Holding p fixed and optimizing over t, we obtain

$$
t^{\mathcal{P}*}(p) = \frac{(\beta+N\Phi)p+\phi d}{(\beta+N\Phi+\phi)p}.
$$

Substituting this back into $\widetilde{\Pi}^{\mathcal{P}*}$, the program becomes

$$
\widetilde{\Pi}^{\mathcal{P}*} = \max_{p}\left\{ N(p-d)(D_0-p) + (p-d)^2\,\Pi^{\mathcal{P}*} \right\},
$$

where $\Pi^{\mathcal{P}*}$ is given by (7). Similarly, solving the program above in $\mathcal{A}$-mode, we have

$$
\widetilde{\Pi}^{\mathcal{A}*} = N\max_{p,t}\left\{ \begin{array}{c} (p-d)(D_0-p) + \frac{N\Phi}{2}(tp-d)((2-t)p-d) \\ +\frac{\phi}{2}p(1-t)((1+t)p-2d) + \frac{1}{2}(1-t)p(\beta-x)((\beta+x+(\beta-x)t)p-2\beta d) \end{array} \right\}.
$$

Holding p fixed and optimizing over t, we obtain

$$
t^{\mathcal{A}*}(p) = \frac{N\Phi p+\phi d+(\beta-x)(\beta d-xp)}{\left(N\Phi+\phi+(\beta-x)^2\right)p}
$$

Substituting this back into $\widetilde{\Pi}^{\mathcal{A}*}$, the program becomes (after straightforward calculations)

$$
\widetilde{\Pi}^{\mathcal{A}*} = \max_{p}\left\{ (p-d)(D_0-p) + (p-d)^2\,\Pi^{\mathcal{A}*} \right\},
$$

where $\Pi^{\mathcal{A}*}$ is given by (9).

Comparing the last expressions of $\widetilde{\Pi}^{\mathcal{A}*}$ and $\widetilde{\Pi}^{\mathcal{P}*}$, we can conclude that

$$\widetilde{\Pi}^{\mathcal{A}*} > \widetilde{\Pi}^{\mathcal{P}*} \Longleftrightarrow \Pi^{\mathcal{A}*} > \Pi^{\mathcal{P}*} \Longleftrightarrow \left|\frac{\phi x}{\beta} + \beta + N\Phi\right| \leq \sqrt{\beta\left(\beta + \phi + N\Phi\right) + \phi^2},$$

so the introduction of p and d does not affect the trade-off determined in Proposition 5 in the main paper.

7 Example with price as transferable decision and linear demand

Recall the revenue function

$$R_i\left(p_i, \overline{p}_{-i}, q_i, Q\right) = p_i\left(d + \beta p_i + x\left(\overline{p}_{-i} - p_i\right) + \phi q_i + \Phi Q\right)$$

and the assumptions made on parameters in the main paper:

$$\begin{aligned} \beta &< 0,\, \phi > 0,\, \Phi > 0 \\ -2\beta + \min\{0, 2x\} &> \max\{N\Phi, \phi\}. \end{aligned} \tag{10}$$

The fixed costs of agents' investment and principal's investment are quadratic:

$$c(q) = \frac{\phi}{2}q^2,\;\; C(Q) = \frac{\Phi}{2}Q^2.$$

In $\mathcal{P}$-mode, the payoff to agent i from working for the principal is

$$(1-t)\, R_i\left(p_i, \overline{p}_{-i}, q_i, Q\right) - \frac{\phi}{2}q_i^2 - T = (1-t)\, p_i\left(d + \beta p_i + x\left(\overline{p}_{-i} - p_i\right) + \phi q_i + \Phi Q\right) - \frac{\phi}{2}q_i^2 - T,$$

which the agent optimizes over q_i in the second stage (the fixed fee T is then taken as fixed).

The principal's payoff in the second stage is

$$\sum_{i=1}^{N}\left(tp_i\left(d + \beta p_i + x\left(\overline{p}_{-i} - p_i\right) + \phi q_i + \Phi Q\right)\right) - \frac{\Phi}{2}Q^2,$$

which the principal optimizes over p_i and Q.

Evaluating the corresponding first-order conditions at the symmetric equilibrium, we have

$$\begin{cases} -2\beta p^{\mathcal{P}} = d + \phi q^{\mathcal{P}} + \Phi Q^{\mathcal{P}} \\ q^{\mathcal{P}} = (1-t)\, p^{\mathcal{P}} \\ Q^{\mathcal{P}} = tNp^{\mathcal{P}}. \end{cases}$$

Solving, we obtain

$$\begin{aligned} p^{\mathcal{P}}(t) &= \tfrac{d}{-2\beta-(1-t)\phi-tN\Phi} \\ q^{\mathcal{P}}(t) &= \tfrac{d(1-t)}{-2\beta-(1-t)\phi-tN\Phi} \\ Q^{\mathcal{P}}(t) &= \tfrac{dtN}{-2\beta-(1-t)\phi-tN\Phi}. \end{aligned}$$

Note that assumptions (10) ensure $p^{\mathcal{P}}(t) > 0$, $q^{\mathcal{P}}(t) > 0$ and $Q^{\mathcal{P}}(t) > 0$.

The fixed fee T is just a transfer that renders each agent indifferent between working for the principal and their outside option, so the principal's profit is

$$\Pi^{\mathcal{P}}(t) = Np^{\mathcal{P}}(t)\left(d + \beta p^{\mathcal{P}}(t) + \phi q^{\mathcal{P}}(t) + \Phi Q^{\mathcal{P}}(t)\right) - N\frac{\phi}{2} q^{\mathcal{P}}(t)^2 - \frac{\Phi}{2} Q^{\mathcal{P}}(t)^2.$$

Plugging in the expressions of $p^{\mathcal{P}}(t)$, $q^{\mathcal{P}}(t)$ and $Q^{\mathcal{P}}(t)$ above, we obtain:

$$\Pi^{\mathcal{P}}(t) = \max_t \left\{ \frac{Nd^2\left(-2\beta - (1-t)^2\phi - t^2 N\Phi\right)}{2\left(-2\beta - (1-t)\phi - tN\Phi\right)^2} \right\}. \tag{11}$$

In $\mathcal{A}$-mode, agent i joining the principal chooses (p_i, q_i) to maximize his second stage payoff

$$(1-t)\, p_i \left(d + \beta p_i + x\left(\overline{p}_{-i} - p_i\right) + \phi q_i + \Phi Q\right) - \frac{\phi}{2} q_i^2,$$

while the principal chooses Q to maximize its second stage revenues

$$\sum_{i=1}^{N} tp_i \left(d + \beta p_i + x\left(\overline{p}_{-i} - p_i\right) + \phi q_i + \Phi Q\right) - \frac{\Phi}{2} Q^2.$$

Evaluating the corresponding first-order conditions at the symmetric equilibrium, we have

$$\begin{cases} (-2\beta + x)\, p^{\mathcal{A}} = d + \phi q^{\mathcal{A}} + \Phi Q^{\mathcal{A}} \\ q^{\mathcal{A}} = (1-t)\, p^{\mathcal{A}} \\ Q^{\mathcal{A}} = tNp^{\mathcal{A}}. \end{cases}$$

Solving, we obtain

$$\begin{aligned} p^{\mathcal{A}}(t) &= \frac{d}{-2\beta+x-(1-t)\phi-tN\Phi} \\ q^{\mathcal{A}}(t) &= \frac{d(1-t)}{-2\beta+x-(1-t)\phi-tN\Phi} \\ Q^{\mathcal{A}}(t) &= \frac{dtN}{-2\beta+x-(1-t)\phi-tN\Phi}. \end{aligned}$$

Assumptions (10) ensure $p^{\mathcal{A}}(t) > 0$, $q^{\mathcal{A}}(t) > 0$ and $Q^{\mathcal{A}}(t) > 0$.

The fixed fee T renders each agent indifferent between joining the principal and his outside option, so the principal's profit in $\mathcal{A}$-mode is

$$\Pi^{\mathcal{A}}(t) = Np^{\mathcal{A}}(t)\left(d + \beta p^{\mathcal{A}}(t) + \phi q^{\mathcal{A}}(t) + \Phi Q^{\mathcal{A}}(t)\right) - N\frac{\phi}{2}q^{\mathcal{A}}(t)^2 - \frac{\Phi}{2}Q^{\mathcal{A}}(t)^2.$$

Plugging in the expressions of $p^{\mathcal{A}}(t)$, $q^{\mathcal{A}}(t)$ and $Q^{\mathcal{A}}(t)$ above, we obtain:

$$\Pi^{\mathcal{A}}(t) = \max_t \left\{ \frac{Nd^2\left(2(-\beta+x) - (1-t)^2\phi - t^2N\Phi\right)}{2(-2\beta+x-(1-t)\phi - tN\Phi)^2} \right\}. \tag{12}$$

Comparing expressions (11) and (12), $\Pi^{\mathcal{P}}(t)$ is obtained from $\Pi^{\mathcal{A}}(t)$ simply by setting $x = 0$. Therefore, we will focus on maximizing $\Pi^{\mathcal{A}}(t)$, from which we can easily derive the maximization of $\Pi^{\mathcal{P}}(t)$.

The first-order derivative of $\Pi^{\mathcal{A}}(t)$ in t is proportional to (with a strictly positive multiplying factor)

$$N\Phi(-2\beta+2x) - \phi x - N\Phi\phi - t\left((N\Phi+\phi)(-2\beta+x) - 2N\Phi\phi\right).$$

Since $(N\Phi+\phi)(-2\beta+x) - 2N\Phi\phi > 0$ under assumptions (10), we obtain that the optimal variable fee under the $\mathcal{A}$-mode is

$$t^{\mathcal{A}*} = \begin{cases} 0 & \text{if } N\Phi(-2\beta+2x) - \phi x - N\Phi\phi \leq 0 \\ \frac{N\Phi(-2\beta+2x)-\phi x - N\Phi\phi}{(N\Phi+\phi)(-2\beta+x)-2N\Phi\phi} & \text{if } 0 \leq N\Phi(-2\beta+2x) - \phi x - N\Phi\phi \\ & \quad \leq (N\Phi+\phi)(-2\beta+x) - 2N\Phi\phi \\ 1 & \text{if } N\Phi(-2\beta+2x) - \phi x - N\Phi\phi \\ & \quad \geq (N\Phi+\phi)(-2\beta+x) - 2N\Phi\phi. \end{cases}$$

Rewriting the conditions:

$$t^{\mathcal{A}*} = \begin{cases} 0 & \text{if } x\left(\phi - 2N\Phi\right) \geq N\Phi\left(-2\beta - \phi\right) \\ \frac{N\Phi(-2\beta+2x)-\phi x - N\Phi\phi}{(N\Phi+\phi)(-2\beta+x)-2N\Phi\phi} & \text{if } x\left(\phi - 2N\Phi\right) \leq N\Phi\left(-2\beta - \phi\right) \\ & \text{and } x\left(N\Phi - 2\phi\right) \leq \phi\left(-2\beta - N\Phi\right) \\ 1 & \text{if } x\left(N\Phi - 2\phi\right) \geq \phi\left(-2\beta - N\Phi\right). \end{cases}$$

Suppose x is such that $0 < t^{\mathcal{A}*} < 1$. Then the first-order condition of $\Pi^{\mathcal{A}}(t)$ in t evaluated at $t^{\mathcal{A}*}$ implies:

$$\left(\left(1 - t^{\mathcal{A}*}\right)\phi - t^{\mathcal{A}*}N\Phi\right)\left(\begin{array}{c} -2\beta + x - \left(1 - t^{\mathcal{A}*}\right)\phi \\ -t^{\mathcal{A}*}N\Phi \end{array}\right) = \left(\phi - N\Phi\right)\left(\begin{array}{c} 2\left(-\beta + x\right) - \left(1 - t^{\mathcal{A}*}\right)^2\phi \\ -\left(t^{\mathcal{A}*}\right)^2 N\Phi \end{array}\right),$$

from which we can deduce:

$$\begin{aligned} \Pi^{\mathcal{A}} &= \frac{Nd^2\left(2\left(-\beta + x\right) - \left(1 - t^{\mathcal{A}*}\right)^2\phi - \left(t^{\mathcal{A}*}\right)^2 N\Phi\right)}{2\left(-2\beta + x - \left(1 - t^{\mathcal{A}*}\right)\phi - t^{\mathcal{A}*}N\Phi\right)^2} \\ &= \frac{Nd^2\left(\left(1 - t^{\mathcal{A}*}\right)\phi - t^{\mathcal{A}*}N\Phi\right)}{2\left(\phi - N\Phi\right)\left(-2\beta + x - \left(1 - t^{\mathcal{A}*}\right)\phi - t^{\mathcal{A}*}N\Phi\right)} \\ &= \frac{Nd^2}{2\left(\phi - N\Phi\right)}\frac{\phi - t^{\mathcal{A}*}\left(N\Phi + \phi\right)}{-2\beta + x - \phi + t^{\mathcal{A}*}\left(\phi - N\Phi\right)}. \end{aligned}$$

Plugging $t^{\mathcal{A}*} = \frac{N\Phi(-2\beta+2x)-\phi x - N\Phi\phi}{(N\Phi+\phi)(-2\beta+x)-2N\Phi\phi}$ into the last expression, we obtain

$$\Pi^{\mathcal{A}*} = \frac{Nd^2}{2}\frac{\left(-2\beta + 2x\right)\left(N\Phi + \phi\right) - N\Phi\phi}{\left(N\Phi + \phi\right)\left(-2\beta - N\Phi + x\right)\left(-2\beta - \phi + x\right) - x\left(N\Phi - \phi\right)^2}.$$

From here, we can set $x = 0$ to obtain

$$\begin{aligned} t^{\mathcal{P}*} &= \frac{\left(-2\beta - \phi\right)N\Phi}{-2\beta\left(N\Phi + \phi\right) - 2N\Phi\phi} \in (0,1) \\ \Pi^{\mathcal{P}*} &= \frac{Nd^2}{2}\frac{-2\beta\left(N\Phi + \phi\right) - N\Phi\phi}{\left(N\Phi + \phi\right)\left(-2\beta - N\Phi\right)\left(-2\beta - \phi\right)}. \end{aligned}$$

The complete characterization of profits in $\mathcal{A}$-mode is:

$$\Pi^{\mathcal{A}*}=\begin{cases}\frac{Nd^2}{2}\frac{2(-\beta+x)-\phi}{(-2\beta+x-\phi)^2} & \text{if } x(\phi-2N\Phi)\geq N\Phi(-2\beta-\phi)\\ \frac{Nd^2}{2}\frac{(-2\beta+2x)(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2} & \text{if } x(\phi-2N\Phi)\leq N\Phi(-2\beta-\phi)\\ & \text{and } x(N\Phi-2\phi)\leq\phi(-2\beta-N\Phi)\\ \frac{Nd^2}{2}\frac{2(-\beta+x)-N\Phi}{(-2\beta+x-N\Phi)^2} & \text{if } x(N\Phi-2\phi)\geq\phi(-2\beta-N\Phi).\end{cases}$$

Suppose $x(\phi-2N\Phi)\leq N\Phi(-2\beta-\phi)$ and $x(N\Phi-2\phi)\leq\phi(-2\beta-N\Phi)$, so that $0<t^{\mathcal{A}*}<1$. We have $\Pi^{\mathcal{A}}>\Pi^{\mathcal{P}}$ if and only if

$$\frac{(-2\beta+2x)(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2}>\frac{-2\beta(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi)(-2\beta-\phi)},$$

which is equivalent to

$$\begin{aligned}&((-2\beta+2x)(N\Phi+\phi)-N\Phi\phi)(N\Phi+\phi)(-2\beta-N\Phi)(-2\beta-\phi)\\ >\;&(-2\beta(N\Phi+\phi)-N\Phi\phi)\left((N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2\right).\end{aligned}$$

Recall the two sides are equal for $x=0$, therefore we can eliminate all terms that are not factored by x or x^2, so the inequality reduces to

$$\begin{aligned}&2x(N\Phi+\phi)^2(-2\beta-N\Phi)(-2\beta-\phi)\\ >\;&(-2\beta(N\Phi+\phi)-N\Phi\phi)\left(-x(N\Phi-\phi)^2+x(N\Phi+\phi)(-4\beta-(N\Phi+\phi))+x^2(N\Phi+\phi)\right).\end{aligned}$$

Rearranging, this can be rewritten

$$\begin{aligned}0\;>\;&-x\begin{pmatrix}(-2\beta(N\Phi+\phi)-N\Phi\phi)\left(2\left(N^2\Phi^2+\phi^2\right)+4\beta(N\Phi+\phi)\right)\\ +2(N\Phi+\phi)^2(-2\beta-N\Phi)(-2\beta-\phi)\end{pmatrix}+\\ &+x^2(-2\beta(N\Phi+\phi)-N\Phi\phi)(N\Phi+\phi).\end{aligned}$$

Simplifying, this leads to

$$0>-2xN\Phi\phi(2\beta(N\Phi+\phi)+2N\Phi\phi)+x^2(-2\beta(N\Phi+\phi)-N\Phi\phi)(N\Phi+\phi),$$

from which we conclude

$$\Pi^{\mathcal{A}*} > \Pi^{\mathcal{P}*} \iff x\left(\frac{2N\Phi\phi\left(-2\beta\left(N\Phi+\phi\right)-2N\Phi\phi\right)}{\left(-2\beta\left(N\Phi+\phi\right)-N\Phi\phi\right)\left(N\Phi+\phi\right)}+x\right) < 0.$$

Both the numerator and the denominator of the large fraction are positive under assumptions (10).

We conclude that when $0 < t^{\mathcal{A}*} < 1$:

$$\begin{aligned}\Pi^{\mathcal{A}*} \quad &> \quad \Pi^{\mathcal{P}*} \iff -\frac{2N\Phi\phi\left(-2\beta\left(N\Phi+\phi\right)-2N\Phi\phi\right)}{\left(-2\beta\left(N\Phi+\phi\right)-N\Phi\phi\right)\left(N\Phi+\phi\right)} < x < 0 \\ &\iff \quad -\frac{4\frac{N\Phi\phi}{N\Phi+\phi}\left(\beta+\frac{N\Phi\phi}{N\Phi+\phi}\right)}{2\beta+\frac{N\Phi\phi}{N\Phi+\phi}} < x < 0\end{aligned}$$

It remains to consider the cases $x\left(\phi-2N\Phi\right) \geq N\Phi\left(-2\beta-\phi\right)$ (in which $t^{\mathcal{A}*}=0$) and $x\left(N\Phi-2\phi\right) \geq \phi\left(-2\beta-N\Phi\right)$ (in which $t^{\mathcal{A}*}=1$). It is easier to consider the following three cases in turn.

Case I: $\phi > 2N\Phi$.

In this case, it is easily verified that assumptions (10) imply $x\left(N\Phi-2\phi\right) \leq \phi\left(-2\beta-N\Phi\right)$. Therefore we have:

$$\Pi^{\mathcal{A}*} = \begin{cases} \frac{Nd^2}{2}\frac{2(-\beta+x)-\phi}{(-2\beta+x-\phi)^2} & \text{if } x \geq \frac{N\Phi(-2\beta-\phi)}{\phi-2N\Phi} \\ \frac{Nd^2}{2}\frac{(-2\beta+2x)(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2} & \text{if } \frac{N\Phi(-2\beta-\phi)}{\phi-2N\Phi} \geq x \geq -\frac{-2\beta-\max\{\phi,N\Phi\}}{2}. \end{cases}$$

The expression $\frac{2(-\beta+x)-\phi}{(-2\beta+x-\phi)^2}$ is increasing in x for $x \leq 0$ and decreasing in x for $x \geq 0$, therefore the maximum value attained by $\Pi^{\mathcal{A}}$ when $x \geq \frac{N\Phi(-2\beta-\phi)}{\phi-2N\Phi}$ is precisely when $x = \frac{N\Phi(-2\beta-\phi)}{\phi-2N\Phi}$. That value is:

$$\begin{aligned}\Pi^{\mathcal{A}*}\left(x=\frac{N\Phi\left(-2\beta-\phi\right)}{\phi-2N\Phi}\right) \quad &= \quad \frac{Nd^2}{2}\frac{\phi\left(\phi-2N\Phi\right)}{\left(-2\beta-\phi\right)\left(\phi-N\Phi\right)^2} \\ &< \quad \frac{Nd^2}{2}\frac{\left(-2\beta\right)\left(N\Phi+\phi\right)-N\Phi\phi}{\left(N\Phi+\phi\right)\left(-2\beta-N\Phi\right)\left(-2\beta-\phi\right)} = \Pi^{\mathcal{P}*},\end{aligned}$$

where the inequality is straightforward to verify under assumptions (10). Thus, $\Pi^{\mathcal{P}*}$ dominates $\Pi^{\mathcal{A}*}$ for all $x \geq \frac{N\Phi(-2\beta-\phi)}{\phi-2N\Phi}$. Combining with the result above, we conclude that $\Pi^{\mathcal{P}*}$ dominates $\Pi^{\mathcal{A}*}$ for all $x \geq 0$ and $x \leq -\frac{4\frac{N\Phi\phi}{N\Phi+\phi}\left(\beta+\frac{N\Phi\phi}{N\Phi+\phi}\right)}{2\beta+\frac{N\Phi\phi}{N\Phi+\phi}}$, whereas $\Pi^{\mathcal{A}*}$ dominates $\Pi^{\mathcal{P}*}$ for all permissible x such that

$$-\frac{4\frac{N\Phi\phi}{N\Phi+\phi}\left(\beta+\frac{N\Phi\phi}{N\Phi+\phi}\right)}{2\beta+\frac{N\Phi\phi}{N\Phi+\phi}} \leq x \leq 0.$$

Case II: $N\Phi > 2\phi$.

In this case, it is easily verified that assumptions (10) imply $x(\phi - 2N\Phi) \leq N\Phi(-2\beta - \phi)$. Therefore we have:

$$\Pi^{\mathcal{A}*} = \begin{cases} \frac{Nd^2}{2}\frac{2(-\beta+x)-N\Phi}{(-2\beta+x-N\Phi)^2} & \text{if } x \geq \frac{\phi(-2\beta-N\Phi)}{N\Phi-2\phi} \\ \frac{Nd^2}{2}\frac{(-2\beta+2x)(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2} & \text{if } \frac{\phi(-2\beta-N\Phi)}{N\Phi-2\phi} \geq x \geq -\frac{-2\beta-\max\{\phi,N\Phi\}}{2}. \end{cases}$$

The analysis is exactly the same as in Case I above (by symmetry in ϕ and $N\Phi$), therefore the conclusion is exactly the same for this case as well.

Case III: $\phi \leq 2N\Phi$ and $N\Phi \leq 2\phi$.

In this case, it is easily verified that assumptions (10) imply $x(N\Phi - 2\phi) \leq \phi(-2\beta - N\Phi)$ and $x(\phi - 2N\Phi) \leq N\Phi(-2\beta - \phi)$ for all permissible x. Therefore we have:

$$\Pi^{\mathcal{A}*} = \frac{Nd^2}{2}\frac{(-2\beta+2x)(N\Phi+\phi)-N\Phi\phi}{(N\Phi+\phi)(-2\beta-N\Phi+x)(-2\beta-\phi+x)-x(N\Phi-\phi)^2}$$

for all permissible x, so we already know that

$$\Pi^{\mathcal{A}*} > \Pi^{\mathcal{P}*} \Longleftrightarrow -\frac{4\frac{N\Phi\phi}{N\Phi+\phi}\left(\beta+\frac{N\Phi\phi}{N\Phi+\phi}\right)}{2\beta+\frac{N\Phi\phi}{N\Phi+\phi}} < x < 0.$$

8 Hybrid mode across agents

Hybrid modes, with some agents offering their services in $\mathcal{P}$-mode and others in $\mathcal{A}$-mode, are found in some of the markets where our theory is relevant (e.g. consultancies, hair salons, and sales representatives for industrial companies). In this section we show that a strictly hybrid mode can be optimal even without spillovers (i.e. we assume $R_\sigma = 0$) and despite the fact that all N agents are identical. This is because Q is a common investment across all the agents' services (e.g. a common infrastructure) and because of the concavity of the profit function with respect to Q.

We use the same linear example from Section 5.1 in the main paper with no spillovers ($x = 0$) and quadratic costs:

$$\begin{aligned} R(a_i, q_i, Q) &= \beta a_i + \phi q_i + \Phi Q \\ f(a_i) &= \frac{\beta}{2}a_i^2,\ c(q_i) = \frac{\phi}{2}q_i^2 \text{ and } C(Q) = \frac{\Phi}{2}Q^2. \end{aligned}$$

At first glance, this seems like the least likely scenario for a hybrid mode to be optimal, because there are no interaction effects and no asymmetries between the principal and the agents. However, it turns out that the principal can find it optimal to use a strictly interior hybrid mode.

Suppose the principal functions in $\mathcal{P}$-mode with respect to agents $i \in \{1,..,n\}$ and in $\mathcal{A}$-mode with respect to agents $i \in \{n+1,..,N\}$, where $n \leq N$. Thus, the principal offers contract $\left(t^{\mathcal{P}}, T^{\mathcal{P}}\right)$ to the n agents that work in $\mathcal{P}$-mode (employees) and contract $\left(t^{\mathcal{A}}, T^{\mathcal{A}}\right)$ to the $N-n$ agents that work in $\mathcal{A}$-mode (independent contractors). The n employees each choose a level of effort equal to $\left(1-t^{\mathcal{P}}\right)$, whereas the $N-n$ independent contractors each choose a level of effort equal to $\left(1-t^{\mathcal{A}}\right)$ and a level of the transferable activity equal to $\left(1-t^{\mathcal{A}}\right)$. For the n employees, the principal chooses a level of the transferable action equal to $t^{\mathcal{P}}$. Finally, the level $Q\left(t^{\mathcal{P}}, t^{\mathcal{A}}\right)$ chosen by the principal is $Q\left(t^{\mathcal{P}}, t^{\mathcal{A}}\right) = \bar{t}N$, where

$$\bar{t} \equiv \frac{n}{N} t^{\mathcal{P}} + \frac{N-n}{N} t^{\mathcal{A}}$$

is the "average" transaction fee collected by the principal.

The fixed fees for employees and independent contractors are set to render both indifferent between working for/through the principal and their outside option. Consequently, the total profit of the principal is

$$\begin{aligned}
\Pi^{H}\left(t^{\mathcal{P}}, t^{\mathcal{A}}, n\right) &= n\left(\frac{t^{\mathcal{P}}\left(2-t^{\mathcal{P}}\right)\beta}{2} + \frac{\left(1-\left(t^{\mathcal{P}}\right)^{2}\right)\phi}{2}\right) + (N-n)\left(\frac{\left(1-\left(t^{\mathcal{A}}\right)^{2}\right)\beta}{2} + \frac{\left(1-\left(t^{\mathcal{A}}\right)^{2}\right)\phi}{2}\right) \\
&\quad + \frac{\bar{t}\left(2-\bar{t}\right)N^{2}\Phi}{2}.
\end{aligned}$$

Optimizing over the three variables $\left(t^{\mathcal{P}}, t^{\mathcal{A}}, n\right)$ yields the following first-order conditions (assuming interior solution in all three variables):

$$\begin{cases}
\beta + N\Phi - \left(\beta + \phi + n\Phi\right) t^{\mathcal{P}} - (N-n)\,\Phi t^{\mathcal{A}} = 0 \\
N\Phi - n\Phi t^{\mathcal{P}} - \left(\beta + \phi + (N-n)\,\Phi\right) t^{\mathcal{A}} = 0 \\
\frac{\beta}{2}\left(t^{\mathcal{P}}\left(2-t^{\mathcal{P}}\right) - 1 + \left(t^{\mathcal{A}}\right)^{2}\right) + \frac{\phi}{2}\left(\left(t^{\mathcal{A}}\right)^{2} - \left(t^{\mathcal{P}}\right)^{2}\right) + N\Phi\left(1-\bar{t}\right)\left(t^{\mathcal{P}} - t^{\mathcal{A}}\right) = 0.
\end{cases}$$

Solving the first two first-order conditions above for $\left(t^{\mathcal{P}}, t^{\mathcal{A}}\right)$ as functions of n, we obtain:

$$\begin{aligned}
t^{\mathcal{P}} &= \tfrac{(\beta+N\Phi)(\beta+\phi)+(N-n)\Phi\beta}{(\beta+\phi)(\beta+\phi+N\Phi)} \\
t^{\mathcal{A}} &= \tfrac{(N-n)\Phi\beta+N\Phi\phi}{(\beta+\phi)(\beta+\phi+N\Phi)}.
\end{aligned}$$

This implies

$$\begin{aligned} t^{\mathcal{P}} - t^{\mathcal{A}} &= \tfrac{\beta}{\beta+\phi} \\ N\left(1-\bar{t}\right) &= \tfrac{(N-n)\beta+N\phi}{\beta+\phi+N\Phi}. \end{aligned}$$

We can now plug these expressions in the third first-order condition above, which becomes:

$$\begin{aligned} \frac{\beta}{2}\left(t^{\mathcal{P}}\left(2-t^{\mathcal{P}}\right)-1+\left(t^{\mathcal{A}}\right)^2\right)+\frac{\phi}{2}\left(\left(t^{\mathcal{A}}\right)^2-\left(t^{\mathcal{P}}\right)^2\right)+\frac{\beta\Phi}{\beta+\phi}\frac{(N-n)\beta+N\phi}{\beta+\phi+N\Phi} &= 0 \\ \frac{\beta}{2}\left(2t^{\mathcal{P}}-1\right)-\frac{\beta+\phi}{2}\frac{\beta}{\beta+\phi}\left(t^{\mathcal{P}}+t^{\mathcal{A}}\right)+\frac{\beta\Phi}{\beta+\phi}\frac{(N-n)\beta+N\phi}{\beta+\phi+N\Phi} &= 0 \\ \frac{1}{2}\left(t^{\mathcal{P}}-t^{\mathcal{A}}-1\right)+\frac{\Phi}{\beta+\phi}\frac{(N-n)\beta+N\phi}{\beta+\phi+N\Phi} &= 0 \\ -\frac{\phi}{2(\beta+\phi)}+\frac{\Phi}{\beta+\phi}\frac{(N-n)\beta+N\phi}{\beta+\phi+N\Phi} &= 0. \end{aligned}$$

The last expression is decreasing in n, which means the second-order condition is satisfied. Solving for n yields

$$n^* = N\left(1-\frac{\phi\left(\beta+\phi-N\Phi\right)}{2N\Phi\beta}\right).$$

This solution is valid if and only if

$$0 < \phi\left(\beta+\phi-N\Phi\right) < 2N\Phi\beta,$$

i.e. if and only if

$$\beta+\phi > N\Phi > \phi-\frac{\beta\phi}{2\beta+\phi}.$$

We can therefore conclude with the following proposition.

Proposition 1 *The optimal number of employees (as opposed to independent contractors) is*

$$n^* = \begin{cases} N & \text{if} \quad N\Phi > \beta+\phi \\ N\left(1-\frac{\phi(\beta+\phi-N\Phi)}{2N\Phi\beta}\right) & \text{if} \quad \beta+\phi > N\Phi > \phi-\frac{\beta\phi}{2\beta+\phi} \\ 0 & \text{if} \quad N\Phi < \phi-\frac{\beta\phi}{2\beta+\phi}. \end{cases}$$

Note that n^* is increasing in $N\Phi$ (the importance of the principal's moral hazard) and decreasing in ϕ (the importance of agents' moral hazard), consistent with the intuition built in Section 5.1 for the case without spillovers.

The reason why the optimal choice of n can be interior (i.e. strictly between 0 and N) is that the principal can only choose a single Q (e.g. infrastructure investment), which affects all agents. If the principal could choose a different Q for agents in each mode, then it would choose a higher Q for agents in $\mathcal{P}$-mode than for agents in $\mathcal{A}$-mode. The optimal level of Q in each mode (and the principal's corresponding profit per agent) would be independent of the number of agents in each mode. The optimal solution would then be $n = N$ or $n = 0$, depending on which mode yields higher profit per agent.

In contrast, when the principal must choose the same Q for agents in both modes, the principal's choice of Q depends on how many agents are in each mode, since it depends on the weighted average of the variable fee in each mode, i.e. $\bar{t}\,(n) \equiv \frac{n}{N}t^{\mathcal{P}} + \frac{N-n}{N}t^{\mathcal{A}}$. Since the principal's profit function is concave in Q, it becomes concave in n (instead of linear): substituting an agent in $\mathcal{A}$-mode for an agent in $\mathcal{P}$-mode has a lower impact on profits when the number of agents in $\mathcal{P}$-mode is larger. This explains why a mix of employees and independent contractors sometimes allows the principal to do better.