# Market Expansion in Duopoly*

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**Abstract**

We study the effect of market expansion due to exogenous factors on a duopoly's equilibrium prices and profits. The total effect on each firm's profits consists of a direct effect, proportional to the demand derived from the new market segment, and a competitive effect, proportional to the change in the rival firm's equilibrium price. We show that the sign of the total effect on profits depends only on first-order and second-order demand elasticities, and on the ratio between how important the new market segment is relative to the existing segments for the two firms. In particular, this sign can be independent of the magnitude of the (positive) direct effect. The total effect on each firm's price consists of a non-strategic effect (the adjustment induced by the demand from the new market segment) and a strategic effect (the response to the price adjustment by the rival firm). Due to the strategic effect, market expansion to a more price elastic segment for both firms can lead to a higher equilibrium price for one firm. We also derive implications for empirical estimations of the effects of market expansion.

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# 1 Introduction

Oligopolies can see their relevant markets expand unexpectedly due to exogenous factors, such as decreases in the price (or increases in the quality) of an underlying platform or a complementary technology, the appearance of new information or cultural trends that change customer perceptions, etc. For example, as videogames gradually entered the cultural mainstream, the addressable market for makers of home videogame consoles (Microsoft, Nintendo, Sega, Sony) expanded from teenage boys in the late 1980s, to older males throughout the 1990s, and finally to just about everyone else after the release of the Nintendo Wii in 2005. Similarly, the cloud infrastructure and computing services offered by Amazon, Google, IBM and Microsoft initially only appealed to start-ups and small companies. Due to the emergence of virtualization and other complementary technologies that helped alleviate security and reliability concerns for cloud services, the relevant market has significantly expanded to also include large corporations.

What is the effect on firm profits of an exogenous market expansion in the absence of new entry? Whereas market expansion always increases monopoly profits, the effect on oligopoly profits can go either way. At a high level, market expansion has both a direct effect and a competitive effect on each firm's profit. The direct effect is always positive (more customers to sell to), while the competitive effect is negative (respectively, positive) if the competitor's new equilibrium price is lower (respectively, higher). If the competitive effect is negative and outweighs the direct effect, market expansion decreases equilibrium profits—we derive precise conditions under which this occurs. Although we focus on a competitive duopoly, our formal analysis and insights also apply when the two firms are complementors (e.g. Microsoft and Intel in the market for personal computers).

In our model, there is a (continuous) set of existing market segments and we conceptualize market expansion as the addition of a new (infinitesimal) market segment. In this set-up, whether market expansion increases or decreases equilibrium prices and profits depends only on the first-order and second-order demand elasticities, and on the ratio between how important the new market segment is relative to the existing segments for the two firms. An important and novel result of our analysis is that the sign of the total effect on profits can be independent of the magnitude of the direct effect, which is proportional to the size of the new market segment and is always positive. For instance, for two symmetric firms, the size of the new market segment has no bearing on whether market expansion decreases or increases equilibrium profits. In fact, if the sign of the profit change is negative, then increasing the new market segment's size exacerbates the negative impact on profits.

Market expansion can also have a counter-intuitive total effect on equilibrium prices. On the one hand, if the new market segment is more price elastic for the two firms, then, assuming no price discrimination, each firm is inclined to decrease its price for the entire market—this is the non-strategic effect. However, there is also a strategic effect: each firm's price decrease affects the competitor's demand elasticity, prompting the competitor to further adjust its price—downward if the two prices are strategic complements, and upward if they are strategic substitutes. Thus, if the prices exhibit sufficiently strong strategic substitutability, the strategic effect may overturn the non-strategic effect, leading to a counter-intuitive result for one of the two firms, i.e. market expansion to a *more price elastic* segment leads to a price *increase*.

Our analysis has important implications for empirical studies. In particular, both first-order and second-order price elasticities must be accurately estimated for the existing market in order to correctly predict the sign of the total effects of market expansion on prices and profits. Furthermore, we show that different sources of consumer heterogeneity affect the outcome differently. Thus, empirical models that accomodate only one type of consumer heterogeneity can yield qualitatively wrong preditions regarding the effect of market expansion.

The next section discusses the related literature. Section 3 sets up our model and derives the expressions for the total effects of market expansion on equilibrium prices and profits. To build intuition for interpreting these expressions, Section 4 analyzes and interprets a simpler version of our model, by shutting down the strategic effect. Section 5 returns to the full equilibrium, interprets the expressions obtained in Section 3, and derives additional results. Section 6 briefly shows that our analysis also applies to complementors. Section 7 illustrates our main results with simple examples based on linear demand. Section 8 discusses the implications of our theoretical analysis for empirical estimations. Finally, Section 9 concludes.

## 2 Related literature

The idea that seemingly beneficial exogenous shocks (i.e. those that would unambiguously raise profits for a monopoly firm) may decrease the equilibrium profits of oligopoly firms even in the absence of entry dates back to at least Bulow et al. (1985). Specifically, Bulow et al. (1985) showed that an exogenous cost decrease in one market for a firm that benefits from economies of scale across two markets can hurt that firm's overall profitability if it induces a sufficiently aggressive competitive response by its rival in the second market. This counter-intuitive phenomenon can only occur if the strategic effect—i.e. the change in profits due to the induced change in the choice of strategic variable (price or quantity) by the competitor in the second market—is negative and outweighs the positive direct effect of the cost decrease in the first market. The sign of the strategic effect is determined by whether the two firms' products are strategic complements or substitutes. In contrast, a key contribution of our analysis is to show that the strategic effect is not essential for obtaining the counter-intuitive result that market expansion can decrease equilibrium profits. The reason is that the exogenous shock in our framework (market expansion) affects both firms directly. This implies that both firms experience a direct effect and a competitive effect, the latter of which can be further decomposed in a strategic effect and a non-strategic effect. The non-strategic effect can deliver the counter-intuitive result on its own. Meanwhile, in Bulow et al. (1985) the exogenous shock is unilateral, i.e. only one firm experiences a direct effect. This implies that the competitive effect is reduced to the strategic effect, which then becomes crucial to the result. Furthermore, Bulow et al. (1985) informally suggest that for the cost decrease to lower total profits, the (positive) direct effect has to be small relative to the strategic effect.[3] By contrast, we completely characterize the necessary and sufficient conditions for market expansion to lower profits and show that in our framework, the sign of the total effect of market expansion on profits can be independent of the magnitude of the (positive) direct effect. In particular, it is possible that increasing the magnitude of the direct effect exacerbates the negative total effect.

Also related is Seade (1985), who shows that in an oligopolistic setting with no entry, an increase in marginal cost common to all competitors (e.g. tax increase) can sometimes lead to higher profit

[3] Specifically, sales in the market where the focal firm experiences the cost decrease must be sufficiently small relative to sales in the oligopolistic market.

for all competitors. While in this set-up all firms face a direct effect, a non-strategic competitive effect and a strategic competitive effect, the mechanism with cost shocks is quite different from the one with market expansion that we study. Furthermore, Seade (1985) relies on a Cournot quantity competition model, whereas we use a general Bertrand competition model, which yields more tractable conditions and is easier to interpret.

Finally, at a high level, our paper is also reminiscent of an earlier literature, which points out that increased competition in an oligopolistic setting (i.e. a higher number of competitors), can result in higher equilibrium prices—see, for example, Salop (1979), Satterthwaite (1979), Rosenthal (1980). We obtain a somewhat similar result, namely that market expansion to a more elastic segment can result in a price increase for one of the competitors, but the underlying mechanisms are very different.

## 3 Set up and derivation of total effects

Consider a duopoly with firms A and B. Throughout sections 3, 4 and 5, the two firms are assumed to be competitors. In section 6, we assume the two firms are complementors (and each of them is a monopolist in its market)—the analysis is almost identical, with the exception of changes in the sign of certain effects.

Demands for the two rival firms are

$$Q^A(p_A, p_B, M) = \int_0^M q^A\left(p_A, p_B, m\right) dm \quad \text{and} \quad Q^B(p_B, p_A, M) = \int_0^M q^B\left(p_B, p_A, m\right) dm,$$

where $M \in [0,1]$ denotes the number of active market segments, and $q^i\left(p_i, p_{-i}, m\right)$ is positive, continuously differentiable in $(p_i, p_{-i})$, decreasing in $p_i$ and increasing in $p_{-i}$ (firms are competitors) for $i \in \{A, B\}$. This implies that $Q^i\left(p_i, p_{-i}, M\right)$ is also positive, continuously differentiable in $(p_i, p_{-i})$, decreasing in $p_i$ and increasing in $p_{-i}$ for $i \in \{A, B\}$. Furthermore, we assume the right derivative of $Q^i\left(p_i, p_{-i}, M\right)$ with respect to $M$ is well-defined for every $M \in [0,1]$, so that

$$\frac{\partial_+ Q^i\left(p_i, p_{-i}, M\right)}{\partial M} = q^i\left(p_i, p_{-i}, M\right)$$

In particular, we allow $q^i(p_i, p_{-i}, M)$ to be discontinuous in $M$. Given that we are interested in market expansion throughout the paper, i.e. an increase in $M$, only the right derivative of $Q^i(p_i, p_{-i}, M)$ in $M$ is relevant. Note also that we have implicitly assumed $m$ is uniformly distributed over $[0,1]$, which is without loss of generality: any density function can be simply folded into the underlying demands $q^A(p_A, p_B, m)$ and $q^B(p_B, p_A, m)$.

Intuitively, the market consists of a continuum of segments $m \in [0,1]$; demands in each segment $m$ are $q^A(p_A, p_B, m)$ and $q^B(p_B, p_A, m)$. Throughout the paper, we are interested in the effect of increasing the number $M$ of active market segments on equilibrium firm profits. This increase can result from an unexpected decrease in the price of an underlying technology or of a complementary product, or from the availability of new information that positively changes customer perceptions. We assume that the firms cannot price discriminate across the different market segments. For simplicity, we also assume both firms have zero marginal costs: in appendix A we show that this assumption involves no loss of substance.

The profits of the two firms are

$$\pi_A = p_A Q^A(p_A, p_B, M) \quad \text{and} \quad \pi_B = p_B Q^B(p_B, p_A, M).$$

The Nash equilibrium with $M$ active market segments is then defined by

$$p_A^*(M) = \underset{p_A}{argmax}\, \pi_A(p_A, p_B^*, M) \quad \text{and} \quad p_B^*(M) = \underset{p_B}{argmax}\, \pi_A(p_A^*, p_B, M).$$

Throughout the paper, we use "*" only for equilibrium prices and profits. We omit "*" for quantities: quantities are always evaluated at the prices that appear in the relevant expressions.

To ensure well-defined maximization problems and stability of the solutions to the simultaneous pricing game, we assume that the following conditions hold at all equilibrium values:

$$\frac{\partial^2 \pi_A}{\partial p_A^2} < 0 \text{ and } \frac{\partial^2 \pi_B}{\partial p_B^2} < 0$$

$$\Delta \equiv \frac{\partial^2 \pi_A}{\partial p_A^2}\frac{\partial^2 \pi_B}{\partial p_B^2} - \frac{\partial^2 \pi_A}{\partial p_A \partial p_B}\frac{\partial^2 \pi_B}{\partial p_A \partial p_B} > 0. \tag{3.1}$$

## 3.1 Decomposing the effect of market expansion

Using the Envelope Theorem for $i \in \{A, B\}$, we can write the effect of an exogenous market expansion (increase in $M$) on equilibrium firm profits as

$$\frac{d\pi_i^*}{dM} = \underbrace{\frac{\partial \pi_i}{\partial M}}_{\text{direct effect}} + \underbrace{\frac{\partial \pi_i}{\partial p_{-i}} \frac{dp_{-i}^*}{dM}}_{\text{competitive effect}} . \tag{3.2}$$

As the market expands, the customer base expands—at the old equilibrium prices, both firms can simply sell more and earn more profit. This is the *direct* effect of market expansion and corresponds to the first term in equation 3.2. It is always positive and can be re-written as $\frac{\partial \pi_i}{\partial M} = p_i q^i$ for $i \in \{A, B\}$.

The second term in equation 3.2 is the *competitive effect* of market expansion. Since $\frac{\partial \pi_i}{\partial p_{-i}} = p_i \frac{\partial Q^i}{\partial p_{-i}} > 0$, the sign of the competitive effect is equal to the sign of $\frac{dp_{-i}^*}{dM}$, i.e. the effect of market expansion on the competitor's price. When the market expands to a demand segment with a different price elasticity from the average elasticity of the existing segments, firms adjust their prices accordingly. For instance, if the new market segment is more price elastic, each firm has an incentive to accommodate the new customers with lower prices, which has a negative effect on the *other* firm's profit. This is the *non-strategic* effect of market expansion on equilibrium prices. However, there is also a *strategic* effect: each firm anticipates the other's price decrease, therefore further adjusts its own price—downward if the two prices are strategic complements (i.e. $\frac{\partial^2 \pi_B}{\partial p_A \partial p_B} > 0$) or upward if they are strategic substitutes (i.e. $\frac{\partial^2 \pi_B}{\partial p_A \partial p_B} < 0$). In the latter case, it is possible that the strategic effect overwhelms the non-strategic effect for one of the two firms, so that firm ends up with a higher equilibrium price as a result of market expansion to a more elastic segment (Corollary 3 below shows that this cannot hold for both firms). For convenience, figure 3.1 summarizes the taxonomy of the various effects of market expansion.

To determine $\frac{dp_i}{dM}$, we start with the first-order conditions $\frac{\partial \pi_A}{\partial p_A} = 0$ and $\frac{\partial \pi_B}{\partial p_B} = 0$ (which must

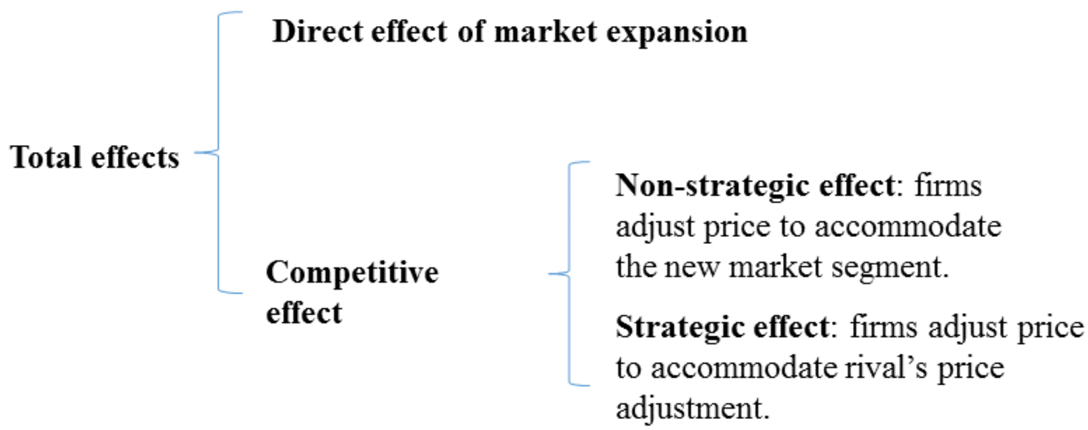


Fig. 3.1: Decomposition of the Total Effect of Market Expansion

hold for all $M$) and take the total derivatives with respect to $M$, obtaining

$$\begin{aligned} \frac{\partial^2 \pi_A}{\partial p_A}\frac{d\,p_A^*}{d\,M} + \frac{\partial^2 \pi_A}{\partial p_A \partial p_B}\frac{d\,p_B^*}{d\,M} + \frac{\partial^2 \pi_A}{\partial p_A \partial M} = 0 \\ \frac{\partial^2 \pi_B}{\partial p_B}\frac{d\,p_B^*}{d\,M} + \frac{\partial^2 \pi_B}{\partial p_A \partial p_B}\frac{d\,p_A^*}{d\,M} + \frac{\partial^2 \pi_B}{\partial p_B \partial M} = 0 \end{aligned} \tag{3.3}$$

Solving these two equations for $\frac{d\,p_A}{d\,M}$ and $\frac{d\,p_B}{d\,M}$ leads to

$$\begin{aligned} \frac{d\,p_A^*}{d\,M} = \frac{1}{\Delta}\left(\frac{\partial^2 \pi_A}{\partial p_A \partial p_B}\frac{\partial^2 \pi_B}{\partial p_B \partial M} - \frac{\partial^2 \pi_B}{\partial p_B^2}\frac{\partial^2 \pi_A}{\partial p_A \partial M}\right) \\ \frac{d\,p_B^*}{d\,M} = \frac{1}{\Delta}\left(\frac{\partial^2 \pi_B}{\partial p_A \partial p_B}\frac{\partial^2 \pi_A}{\partial p_A \partial M} - \frac{\partial^2 \pi_A}{\partial p_A^2}\frac{\partial^2 \pi_B}{\partial p_B \partial M}\right), \end{aligned} \tag{3.4}$$

where $\Delta > 0$ is the determinant defined in equation 3.1. The term $\frac{\partial^2 \pi_i}{\partial p_i \partial M}$ in the expression of $\frac{d\,p_i^*}{d\,M}$ corresponds to the non-strategic price effect described above, whereas the term $\frac{\partial^2 \pi_i}{\partial p_i \partial p_{-i}}\frac{\partial^2 \pi_{-i}}{\partial p_{-i} \partial M}$ corresponds to the strategic price effect.

## 3.2 Elasticities

We will characterize the effect of market expansion on equilibrium prices and profits in terms of elasticities. To do so, we need to introduce the following notation (for conciseness, we omit the arguments $(p_A, p_B, M)$):

| | | |
|---|---|---|
| average (first-order) self elasticity | $\bar{\varepsilon}_A^A = -\frac{\partial \log(Q^A)}{\partial \log(p_A)}$ | positive |
| marginal (first-order) self elasticity | $\varepsilon_A^A = -\frac{\partial \log(q^A)}{\partial \log(p_A)} > 0$ | positive |
| (first-order) cross elasticity | $\bar{\varepsilon}_B^A = \frac{\partial \log(Q^A)}{\partial \log(p_B)} > 0$ | positive for competitors; negative for complementors |
| second-order self elasticity | $\varepsilon_A^{\bar{\varepsilon}_A^A} = \frac{\partial \log(\bar{\varepsilon}_A^A)}{\partial \log(p_A)}$ | positive |
| second-order cross elasticity | $\varepsilon_B^{\bar{\varepsilon}_A^A} = -\frac{\partial \log(\bar{\varepsilon}_A^A)}{\partial \log(p_B)}$ | positive for strategic substitutes; negative otherwise |

Tab. 1: First-order and Second-order Elasticities

Thus, $\bar{\varepsilon}_A^A$ and $\bar{\varepsilon}_B^B$ are the first-order own-price demand elasticities for firms A and B in the *existing* market, whereas $\varepsilon_A^A$ and $\varepsilon_B^B$ are the own-price elasticities faced by A and B in the *new* market segment. The first-order own-price elasticities determine the profit-maximizing prices: with zero marginal cost, the first-order condition in price is equivalent to setting the own-price elasticity equal to 1. Meanwhile, second-order elasticities (last two rows in Table 1 ) determine how much the first-order conditions change with price. For example, A's second-order self elasticity $\left(\varepsilon_A^{\bar{\varepsilon}_A^A}\right)$ quantifies A's ability to move its own first-order condition by changing its price. The larger this elasticity is, the less price adjustment is necessary for A to restore its first-order condition in response to an exogenous shock (e.g. market expansion).

The signs of the elasticities are implied by the assumptions made earlier on the demand functions and by the following lemma (all proofs are in the appendix).

**Lemma 1.** *For any $M$,* at the price equilibrium $\left(p_A^*(M), p_B^*(M)\right)$ *we have*

$$\frac{\partial^2 \pi_A}{\partial p_A^2} = -\frac{Q^A}{p_A^*}\varepsilon_A^{\bar{\varepsilon}_A^A} \quad \textit{and} \quad \frac{\partial^2 \pi_B}{\partial p_B^2} = -\frac{Q^B}{p_B^*}\varepsilon_B^{\bar{\varepsilon}_B^B} \tag{3.5}$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = \frac{Q^A}{p_B^*}\varepsilon_B^{\bar{\varepsilon}_A^A} \quad \textit{and} \quad \frac{\partial^2 \pi_B}{\partial p_A \partial p_B} = \frac{Q^B}{p_A^*}\varepsilon_A^{\bar{\varepsilon}_B^B} \tag{3.6}$$

$$\frac{\partial^2 \pi_A}{\partial p^A \partial M} = q^A(1-\varepsilon_A^A) \quad \textit{and} \quad \frac{\partial^2 \pi_B}{\partial p^B \partial M} = q^B(1-\varepsilon_B^B). \tag{3.7}$$

The expressions in 3.5 imply that for second-order conditions to hold at the profit-maximizing prices, the second-order self elasticities must be positive, i.e.[4]

$$\varepsilon_A^{\bar{\varepsilon}_A^A} > 0 \quad \text{and} \quad \varepsilon_B^{\bar{\varepsilon}_B^B} > 0.$$

This is equivalent to Marshall's Second Law of Demand.[5]

Expression 3.6 implies that the signs of the second-order cross elasticities evaluated at equilibrium determine whether the prices of the two goods are *strategic* substitutes or strategic complements for each of the two firms. Specifically, if increasing A's price makes B's demand more elastic, then B decreases its equilibrium price, so B views the price of A as a strategic substitute to its own price. By contrast, the signs of the first-order elasticities $\bar{\varepsilon}_B^A$ and $\bar{\varepsilon}_A^B$ determine whether the two firms' products are *simple* substitutes or complements: here, the two firms are competitors, so $\bar{\varepsilon}_B^A > 0$ and $\bar{\varepsilon}_A^B > 0$, whereas in section 6 we analyze the case in which they are complementors, so $\bar{\varepsilon}_B^A < 0$ and $\bar{\varepsilon}_A^B < 0$.

Finally, to interpret equation 3.7 in lemma 1, recall that at the price equilibrium $\bar{\varepsilon}_A^A = 1$ and $\bar{\varepsilon}_B^B = 1$. Thus, when the market expands to a more elastic segment for firm A $\left(\varepsilon_A^A > 1 = \overline{\varepsilon_A^A}\right)$, the average demand elasticity faced by A increases, making it more profitable for firm A to decrease price—this captures the non-strategic effect of market expansion on price.

We can use Lemma 1 to write the stability condition 3.1 as a function of elasticities:

$$\Delta > 0 \iff \varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B} > \varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}. \tag{3.8}$$

[4] Our assumptions of zero marginal costs and well-defined profit maximization imply that the demand functions we consider cannot have constant elasticity, which in turn implies $\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B} \neq 0$. In Appendix A, we show how our model can be modified to accomodate positive marginal costs: in that case, demand functions with constant elasticity are possible.

[5] To understand this condition, note that at the profit-maximizing price, each firm's own price elasticity is equal to 1 (due to the first-order condition). From here, price optimality implies that if a firm increases its price by 1 percent, then its demand must decrease by more than 1 percent, so that the effective price elasticity associated with this price increase must be greater than 1. Similarly, the effective price elasticity associated with a small price decrease must be smaller than 1. By continuity, this implies the slope of the own-price elasticity around the equilibrium price must be positive.

## 3.3 Deriving the total effects of market expansion

The total effects of market expansion on equilibrium prices and profits can be written as functions of the first-order and second-order elasticities. To do so, we replace the various terms that determine $\frac{dp_A^*}{dM}$ and $\frac{dp_B^*}{dM}$ in equation 3.4 with their expressions as functions of elasticities from Lemma 1, then use equation 3.2 (the full derivations are in Appendix C).

**Proposition 1.** *The effect of market expansion on equilibrium prices is*

$$\frac{dp_i^*}{dM} = p_i^* \frac{\frac{q^i}{Q^i} \varepsilon_{-i}^{\overline{\varepsilon}_{-i}^{-i}} \left(1 - \varepsilon_i^i\right) + \frac{q^{-i}}{Q^{-i}} \varepsilon_{-i}^{\overline{\varepsilon}_i^i} \left(1 - \varepsilon_{-i}^{-i}\right)}{\varepsilon_i^{\overline{\varepsilon}_i^i} \varepsilon_{-i}^{\overline{\varepsilon}_{-i}^{-i}} - \varepsilon_i^{\overline{\varepsilon}_{-i}^{-i}} \varepsilon_{-i}^{\overline{\varepsilon}_i^i}} \tag{3.9}$$

*and the effect of market expansion on equilibrium profits is*

$$\frac{d\,\pi_i^*}{d\,M} = p_i^* q^i \left(1 + \overline{\varepsilon}_{-i}^i \frac{\frac{Q^i}{q^i} \frac{q^{-i}}{Q^{-i}} \varepsilon_i^{\overline{\varepsilon}_i^i} \left(1 - \varepsilon_{-i}^{-i}\right) + \varepsilon_i^{\overline{\varepsilon}_{-i}^{-i}} \left(1 - \varepsilon_i^i\right)}{\varepsilon_i^{\overline{\varepsilon}_i^i} \varepsilon_{-i}^{\overline{\varepsilon}_{-i}^{-i}} - \varepsilon_i^{\overline{\varepsilon}_{-i}^{-i}} \varepsilon_{-i}^{\overline{\varepsilon}_i^i}}\right), \tag{3.10}$$

*where* $i \in \{A, B\}$ *and* $-i \equiv \{A, B\} \setminus \{i\}$.

Before providing the full interpretation of these expressions in Section 5, it is useful to discuss a simplified scenario, which is significantly easier to interpret. This also allows us to build intuition that will help in explaining the results for the full equilibrium.

# 4 Myopic play

Myopic play assumes away the strategic effect: in response to an exogenous market expansion (increase in $M$), each firm is assumed to adjust its price taking the other firm's existing price as given. In other words, firms ignore each other's pricing response to market expansion, so this is not a Nash equilibrium. Nevertheless, there are two reasons for studying this scenario. First, myopic play is used in game theory to model learning[6]—it is an intermediate step on the adjustment path

[6] This is a special case of the "fictitious play" introduced by Brown (1951) to model learning in games. One interpretation of fictitious play is a "belief-based" learning rule: in each period, players assume opponents play stationary mixed strategies, form beliefs about opponents' strategies, and behave rationally with respect to these beliefs. Myopic play corresponds to the special case in which every player forms beliefs solely based on the opponents' play in the previous period.

from the pre-expansion Nash equilibrium to the post-expansion Nash equilibrium. Second, this scenario allows us to shut down the strategic component of the competitive effect and focus on the non-strategic component. Using it as a benchmark, we analyze how the strategic effect interacts with the direct effect of market expansion in Section 5.

Formally, the firms respond to a slight market expansion $dM$ by setting prices as follows:

$$p_i^{myopic}(M, dM) = \underset{p_i}{argmax}\, \pi_A(p_i, p_{-i}^*(M), M + dM)$$

Here, the system of equations 3.3 on page 8 still applies, except that we substitute zero for $\frac{\partial^2 \pi_A}{\partial p_A \partial p_B}$ and $\frac{\partial^2 \pi_B}{\partial p_A \partial p_B}$, i.e. each firm ignores the other firm's price change. With a small abuse of differentiation notation, we have

$$\frac{d\,p_i^{myopic}}{d\,M} \equiv \lim_{dM \to 0} \frac{p_i^{myopic}(M, dM) - p_i^*(M)}{dM} = -\frac{\frac{\partial^2 \pi_i}{\partial p_i \partial M}}{\frac{\partial^2 \pi_i}{\partial p_i^2}}.$$

We obtain the following corollary to Proposition 1 (the expressions below are derived by setting the cross-price elasticities of the own-price elasticities $\varepsilon_{-i}^{\bar{\varepsilon}_i^i}$ in equations 3.9 to zero).

**Corollary 1.** *If firms behave myopically, the effect of market expansion on the price for firm $i \in \{A, B\}$ is*

$$\frac{dp_i^{myopic}}{dM} = p_i^* \frac{q^i}{Q^i} \frac{1 - \varepsilon_i^i}{\varepsilon_i^{\bar{\varepsilon}_i^i}} \tag{4.1}$$

*and the effect of market expansion on profits for firm $i \in \{A, B\}$ is*

$$\frac{d\pi_i^{myopic}}{d\,M} = p_i^* q^i \left(1 + \frac{q^{-i} Q^i}{Q^{-i} q^i}\, \bar{\varepsilon}_{-i}^i \frac{1 - \varepsilon_{-i}^{-i}}{\varepsilon_{-i}^{\bar{\varepsilon}_{-i}^{-i}}}\right) \tag{4.2}$$

Let us first interpret the price effect in (4.1): how does market expansion affect firms' prices under myopic play? Suppose the new market segment is more price elastic than the existing market for firm A (i.e. $\varepsilon_A^A(M) > \bar{\varepsilon}_A^A(M) = 1$), so firm A's overall demand elasticity increases. The magnitude of this increase is proportional to the size of the excess elasticity, $\left(\varepsilon_A^A - 1\right)$, weighted by the importance of the new market segment, $\frac{q^A}{Q^A}$. Faced with higher overall demand elasticity, firm

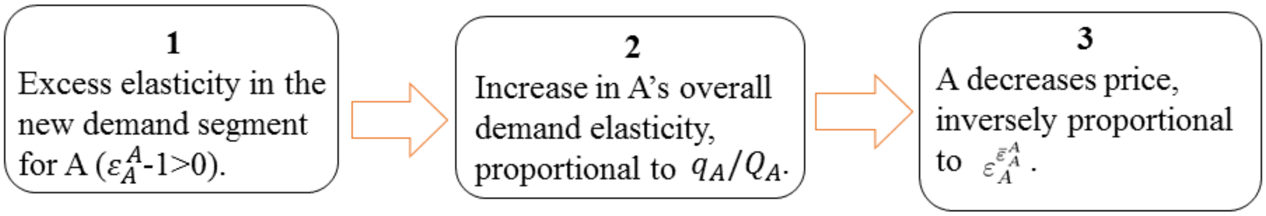


Fig. 4.1: Price Effect of Myopic Play Following Entry into New Market Segment

A finds it optimal to decrease its price. The magnitude of the price decrease necessary to bring its own-price elasticity back to unity (required by profit-maximization with 0 marginal costs) is inversely proportional to the second-order self elasticity $\varepsilon_A^{\bar{\varepsilon}_A^A}$. The logic of this chain of effects is summarized in Figure 4.1.

Consequently, the sign of the myopic price effect (given by 4.1) is solely determined by the own-price elasticity of demand coming from the new market segment. Indeed, since $\varepsilon_i^{\bar{\varepsilon}_i^i}$ is positive, the sign of $\frac{dp_i^{myopic}}{dM}$ is equal to the sign of $1 - \varepsilon_i^i$ for firm $i \in \{A, B\}$. Thus, in the absence of the strategic effect, market expansion causes prices to drop if and only if the marginal segment is more price elastic than the average infra-marginal segments (the average elasticity is equal to 1 at the profit-maximizing prices).

Let us now turn to the effect of market expansion on profits, given by (4.2), which we rewrite here from firm B's perspective :

$$\frac{d\pi_B^{myopic}}{d\,M} = \underbrace{p_B^* q^B}_{\text{direct effect}} \left( 1 + \underbrace{\bar{\varepsilon}_A^B \frac{Q^B}{q^B} \frac{q^A}{Q^A} \frac{1 - \varepsilon_A^A}{\varepsilon_A^{\bar{\varepsilon}_A^A}}}_{\frac{\text{competitive effect}}{\text{direct effect}}} \right). \tag{4.3}$$

Under myopic play, firm B's profit change depends only on the *rival firm* A's demand elasticity in the new market (i.e. $\varepsilon_A^A$), but not on B's own elasticity in the new market (i.e. $\varepsilon_B^B$). Intuitively, in the absence of the strategic effect, only A's demand elasticity in the new market matters for A's price change and therefore for the competitive effect on firm B's profits. Thus, the addition of a more price-elastic segment drags the two firms into a prisoner's dilemma.

The competitive effect on B's profits is also proportional to $\bar{\varepsilon}_A^B > 0$, the elasticity of firm B's profits with respect to firm A's price, and to $\frac{q^A}{Q^A}\frac{Q^B}{q^B}$, which we will henceforth refer to as the "bias of market expansion." When both firms' total demands increase in the same proportion ($\frac{q^A}{Q^A} = \frac{q^B}{Q^B}$), we say that market expansion is neutral. When firm A's demand increases by a higher percentage than B's ($\frac{q^A}{Q^A} > \frac{q^B}{Q^B}$), we say that the market expansion is biased in favor of firm A. Thus, a larger elasticity of B's profits with respect to A's price or a larger bias in favor of firm A amplify firm A's incentive to modify its price and therefore the *competitive* effect of market expansion on B's profits (which is negative if and only if $\varepsilon_A^A > 1$).

An important implication is that, having fixed the bias of market expansion $\frac{q^A}{Q^A}\frac{Q^B}{q^B}$, the sign of the total effect of market expansion on firm profits $\frac{d\pi_B}{dM}$ is independent of the magnitude of the direct effect $p_B^* q^B$. This is most clearly illustrated when the two firms' demand functions are "almost symmetric."

**Definition 1.** *The two firms' demand functions are almost symmetric if there exists a constant $k$, such that for any $(p_1, p_2, M)$,*

$$Q^A(p_A = p_1, p_B = p_2, M) = k\, Q^B(p_B = p_1, p_A = p_2, M).$$

Almost symmetry means the two firms face isomorphic demands. This implies the following:

- For any $M$, there exists a symmetric price equilibrium, i.e. one in which $p_A^*(M) = p_B^*(M)$.
- At the symmetric price equilibrium, the ratio between the respective changes in the two firms' total demands due to market expansion is the same as the ratio between the respective demands, i.e. $\frac{dQ^A}{dM} = k\,\frac{dQ^B}{dM}$.
- The first-order and second-order elasticities for the two firms are identical, i.e. for any triplet $(p_1, p_2, m)$,

$$\begin{cases} \varepsilon_A^A(p_A = p_1, p_B = p_2, m) = & \varepsilon_B^B(p_B = p_1, p_A = p_2, m) \\ \varepsilon_A^B(p_A = p_1, p_B = p_2, m) = & \varepsilon_B^A(p_B = p_1, p_A = p_2, m) \\ \varepsilon_A^{\varepsilon_A^A}(p_A = p_1, p_B = p_2, m) = & \varepsilon_B^{\varepsilon_B^B}(p_B = p_1, p_A = p_2, m) \\ \varepsilon_B^{\varepsilon_A^A}(p_A = p_1, p_B = p_2, m) = & \varepsilon_A^{\varepsilon_B^B}(p_B = p_1, p_A = p_2, m). \end{cases}$$

- Market expansion is always neutral, that is $\frac{q^A}{Q^A}\frac{Q^B}{q^B} = 1$.

**Corollary 2.** *If the two firms behave myopically and their demands are almost symmetric, then the effect of market expansion on profits is*

$$\frac{d\pi_i^*}{d\,M} = p_i^* q^i \left(1 + \bar{\varepsilon}^i_{-i} \frac{1 - \varepsilon^{-i}_{-i}}{\frac{\bar{\varepsilon}^{-i}_{-i}}{\varepsilon^{-i}_{-i}}}\right).$$

*Proof.* The first statement follows directly from **4.3** by setting $q^A = kq^B$ and $Q^A = kQ^B$. □

When the two firms are almost symmetric, they assign the new market segment the same importance, so the sign of the total effect on firm profits is solely determined by elasticities, not by market sizes. In particular, the magnitude of the *direct* effect on firm $i$'s profits, $p_i^* q^i$, has no impact on the sign of the *total* effect of market expansion on firm $i$'s profits. Consequently, if the sign of the total effect is negative (because the new market segment is very price elastic and the elasticity of substitution between the two firms' products is sufficiently large), then an increase in the magnitude of the direct effect (which is always positive) makes the total effect even more negative! This somewhat surprising feature is not an artifact of the myopic play assumed in this section—it also holds in the "proper" Nash equilibrium determined by Proposition 1, as we discuss in the next section.

## 5 Full equilibrium

We now return to the Nash equilibrium and interpret the expressions obtained in Proposition 1.

## 5.1 Effect on equilibrium prices

Consider first the effect of market expansion on equilibrium prices from expression 3.9, which we reproduce here for firm A:

$$\frac{\frac{dp_A^*}{dM}}{p_A^*} = \underbrace{\frac{\varepsilon_A^{\bar\varepsilon_A^A}\varepsilon_B^{\bar\varepsilon_B^B}}{\varepsilon_A^{\bar\varepsilon_A^A}\varepsilon_B^{\bar\varepsilon_B^B} - \varepsilon_A^{\bar\varepsilon_B^B}\varepsilon_B^{\bar\varepsilon_A^A}}}_{\text{iteration factor}} \left( \underbrace{\frac{q^A}{Q^A}\frac{\left(1-\varepsilon_A^A\right)}{\varepsilon_A^{\bar\varepsilon_A^A}}}_{\text{myopic effect}} + \underbrace{\frac{\varepsilon_B^{\bar\varepsilon_A^A}}{\varepsilon_A^{\bar\varepsilon_A^A}}\frac{q^B}{Q^B}\frac{\left(1-\varepsilon_B^B\right)}{\varepsilon_B^{\bar\varepsilon_B^B}}}_{\text{strategic effect}} \right) \tag{5.1}$$

The myopic effect was isolated in the previous section (cf. expression 4.1) and it captures firm A's direct reaction to market expansion, not taking into account firm B's reaction. The strategic price effect of market expansion was entirely shut down in the previous section: given that firm B also reacts directly to market expansion, firm A further adjusts its own price in response to the anticipated change in firm B's price. The sign of the myopic effect is solely determined by the excess price elasticity of demand for firm A coming from the new market segment, whereas the magnitude is proportional to the relative importance of the new market segment for firm A (just like in the previous section). In contrast, the sign of the strategic price effect is determined by the excess elasticity of demand for the rival firm B $(1-\varepsilon_B^B)$ and by whether the prices of A and B are strategic complements or substitutes from firm A's perspective $(\varepsilon_B^{\bar\varepsilon_A^A})$. The magnitude of the strategic effect is proportional to the relative importance of the new market segment for firm B.

To understand the iteration factor in 5.1, note that the new price equilibrium after market expansion can be seen as resulting from infinite iteration of myopic play in the following way. In the first iteration, each firm only responds to market expansion and therefore the firms increase their prices by $dp_A^1 = p^A \frac{q^A}{Q^A}\frac{1-\varepsilon_A^A}{\varepsilon_A^{\bar\varepsilon_A^A}} dM$ and $dp_B^1 = p^B \frac{q^B}{Q^B}\frac{1-\varepsilon_B^B}{\varepsilon_B^{\bar\varepsilon_B^B}} dM$ (these are the price changes under myopic play from 4.1). In the second iteration, each firm observes that its rival's price change has increased its overall demand elasticity by $d\bar\varepsilon_A^A = -\varepsilon_B^{\bar\varepsilon_A^A}\frac{dp_B^1}{p_B}\bar\varepsilon_A^A$ and $d\bar\varepsilon_B^B = -\varepsilon_A^{\bar\varepsilon_B^B}\frac{dp_A^1}{p_A}\bar\varepsilon_B^B$ . In response, the firms further adjust their prices by

$$dp_A^2 = -\frac{d\bar\varepsilon_A^A/\bar\varepsilon_A^A}{\varepsilon_A^{\bar\varepsilon_A^A}} p_A = p_A \frac{q^B}{Q^B}\frac{1-\varepsilon_B^B}{\varepsilon_B^{\bar\varepsilon_B^B}}\frac{\varepsilon_B^{\bar\varepsilon_A^A}}{\varepsilon_A^{\bar\varepsilon_A^A}} dM \text{ and } dp_B^2 = -\frac{d\bar\varepsilon_B^B/\bar\varepsilon_B^B}{\varepsilon_B^{\bar\varepsilon_B^B}} p_B = p_B \frac{q^A}{Q^A}\frac{1-\varepsilon_A^A}{\varepsilon_A^{\bar\varepsilon_A^A}}\frac{\varepsilon_A^{\bar\varepsilon_B^B}}{\varepsilon_B^{\bar\varepsilon_B^B}} dM$$

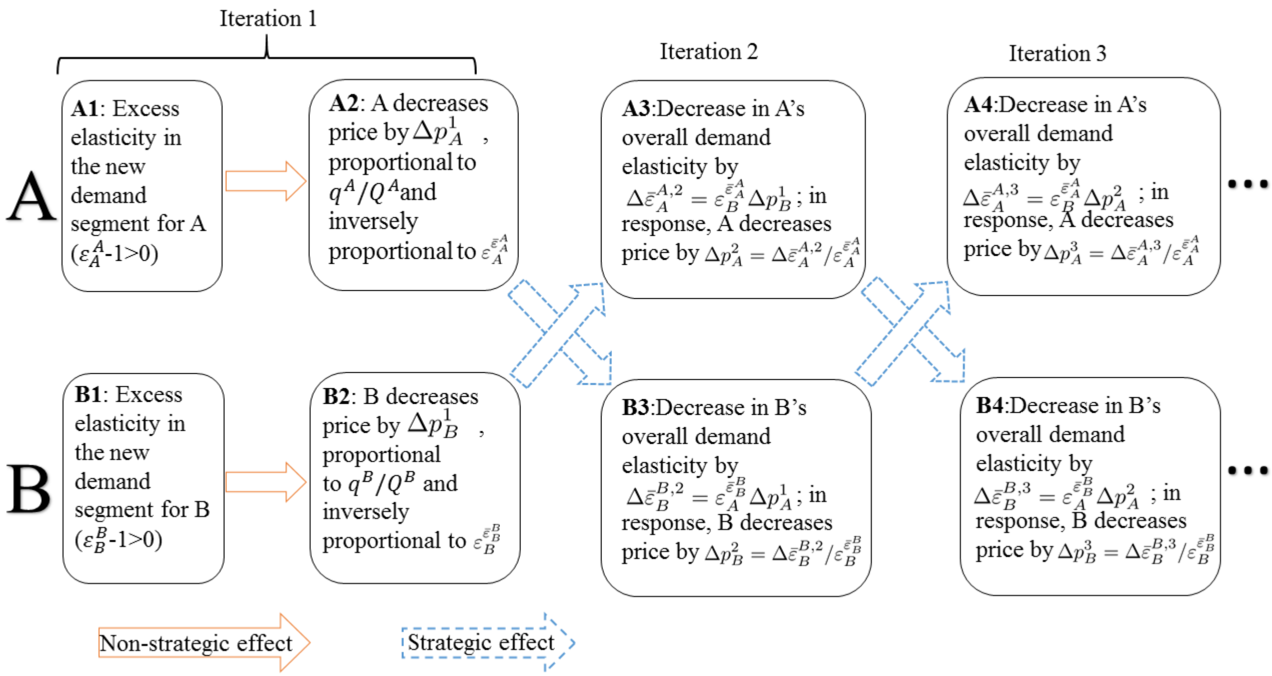


Fig. 5.1: Price adjustment iterations following expansion to new market segment

in order to bring their respective overall demand elasticities back to 1. Indeed, $\varepsilon_B^{\bar{\varepsilon}_A^A} > 0$ implies the prices of A and B are strategic complements for A, so an increase in $p_B$ leads A to also increase its price. This in turn leads to further adjustments of $dp_A^3 = p_A \frac{q^A}{Q^A} \frac{1-\varepsilon_A^A}{\varepsilon_A^{\bar{\varepsilon}_A^A}} \left( \frac{\varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B}} \right) dM$ and $dp_B^3 = p_B \frac{q^B}{Q^B} \frac{1-\varepsilon_B^B}{\varepsilon_B^{\bar{\varepsilon}_B^B}} \left( \frac{\varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B}} \right) dM$ in the third iteration and $dp_A^4 = p_A \frac{q^B}{Q^B} \frac{1-\varepsilon_B^B}{\varepsilon_B^{\bar{\varepsilon}_B^B}} \frac{\varepsilon_A^{\bar{\varepsilon}_B^B}}{\varepsilon_A^{\bar{\varepsilon}_A^A}} \left( \frac{\varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B}} \right) dM$ and $dp_B^4 = p_B \frac{q^A}{Q^A} \frac{1-\varepsilon_A^A}{\varepsilon_A^{\bar{\varepsilon}_A^A}} \frac{\varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_B^{\bar{\varepsilon}_B^B}} \left( \frac{\varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B}} \right) dM$ in the fourth iteration. And so on. Since $\left| \frac{\varepsilon_A^{\bar{\varepsilon}_B^B} \varepsilon_B^{\bar{\varepsilon}_A^A}}{\varepsilon_A^{\bar{\varepsilon}_A^A} \varepsilon_B^{\bar{\varepsilon}_B^B}} \right| < 1$ due to condition 3.8, the amplitude of this sequence of iterations converges to 0 and the total resulting price change for firm A is easily verified to be equal to the expression in 5.1.[7] And similarly for firm B. Figure 5.1 summarizes the effect of market expansion on prices through the iterations described above.

An interesting consequence of the strategic price effect is that, even when the new market segment is more price elastic for both firms (i.e. $\varepsilon_i^i > 1$ for $i \in \{A, B\}$), one of the new equilibrium prices might be higher. Indeed, suppose the prices of A and B are strategic substitutes from the perspective of A's profits, i.e. $\varepsilon_B^{\bar{\varepsilon}_A^A} < 0$. This means that firm B's propensity to decrease its price

[7] Specifically, the sum of the odd-numbered (respectively, even-numbered) iterations converges to the myopic effect (respectively, strategic effect) multiplied by the iteration factor.

due to market expansion leads firm A to increase its price through the strategic effect. If this strategic effect is sufficiently strong, then it may overwhelm the non-strategic (myopic) effect and A's new price might end up being higher than before. However, this cannot hold for both firms at the same time due to the stability condition 3.8 on page 10. This result is formalized in the following corollary.

**Corollary 3.** *If the new market segment is more price elastic than the existing market for both firms (i.e. if $\varepsilon_B^B > 1$ and $\varepsilon_A^A > 1$) then at least one of the two prices must decrease as a result of market expansion, i.e.* $\frac{dp_A^*}{dM} < 0$ *or* $\frac{dp_B^*}{dM} < 0$.

## 5.2 Effect on equilibrium profits

Let us now turn to the discussion of the total effect of market expansion on firm profits, given by expression 3.10, which we reproduce here for firm A:

$$\frac{d\pi_A^*}{dM} = p_A^* q_A \left( 1 + \bar{\varepsilon}_B^A \frac{\frac{Q^A}{q^A}\frac{q^B}{Q^B}\varepsilon_A^{\bar{\varepsilon}_A^A}\left(1 - \varepsilon_B^B\right) + \varepsilon_A^{\bar{\varepsilon}_B^B}\left(1 - \varepsilon_A^A\right)}{\varepsilon_A^{\bar{\varepsilon}_A^A}\varepsilon_B^{\bar{\varepsilon}_B^B} - \varepsilon_A^{\bar{\varepsilon}_B^B}\varepsilon_B^{\bar{\varepsilon}_A^A}} \right) \tag{5.2}$$

Compared to the case of myopic play (expression 4.2), the main difference is that now, the sign of the total effect on A's profits also depends on the demand elasticity of the new market segment for firm A, i.e. $\varepsilon_A^A$. By contrast, in the case of myopic play, only the rival's new demand elasticity, $\varepsilon_B^B$, mattered for the sign of $\frac{d\pi_A}{dM}$. Here, due to the strategic effect, how much firm B changes its price also depends on how much firm A changes its price, which in turn rests on the elasticity of the new market for firm A. An interesting consequence is that, even if firm A enters a more elastic new market ($\varepsilon_A^A > 1$), this does not necessarily mean that the competitive effect will work against it (recall the competitive effect is the sum of the myopic effect and of the strategic effect). Specifically, if the prices of A and B are strategic substitutes from B's perspective ($\varepsilon_A^{\bar{\varepsilon}_B^B} < 0$), then the more elastic the new market for A, the stronger the favorable strategic effect: A's price-cutting to accommodate the new market segment makes B's demand less elastic on average, prompting B to increase its price, or at least to reduce the amount by which it cuts its price in direct response to expansion into a more elastic market.

Intuitively, looking at the initial decomposition of the total effect on firm profits given by expression 3.2 on page 7, one would expect that the direct effect of market expansion should determine both the magnitude and the sign of the total effect, i.e. a larger direct effect should make the total effect more likely to be positive, as in Bulow et al. (1985). However, this need not be the case in our model. Expression 5.2 shows that if the ratio $\frac{Q^A}{q^A}\frac{q^B}{Q^B}$ is held constant, then the sign of the total effect $\frac{d\pi_A}{dM}$ is independent of the magnitude of the direct effect $p_A q_A$. This is most clearly seen when the two firms are almost symmetric.

**Corollary 4.** *If the two firms are almost symmetric, then the effect of market expansion on equilibrium prices is*

$$\frac{dp_i^*}{dM} = p_i^* \frac{q^i}{Q^i} \frac{1 - \varepsilon_i^i}{\varepsilon_i^{\bar{\varepsilon}_i^i} - \varepsilon_i^{\bar{\varepsilon}_{-i}^{-i}}} \tag{5.3}$$

*and the total effect on profits is*

$$\frac{d\pi_i^*}{dM} = p_i^* q^i \left( 1 + \bar{\varepsilon}_{-i}^i \frac{1 - \varepsilon_{-i}^{-i}}{\varepsilon_{-i}^{\bar{\varepsilon}_{-i}^{-i}} - \varepsilon_{-i}^{\bar{\varepsilon}_i^i}} \right), \tag{5.4}$$

*Proof.* These expressions are derived directly from (3.9) and (3.10) by imposing almost symmetry.

□

With almost symmetric firms, the sign of the change in firm $i$'s equilibrium price – and therefore of the competitive effect in 5.4 – is once again equal to the sign of $\left(1 - \varepsilon_{-i}^{-i}\right)$, just like under myopic play (recall 4.1). The only difference is the term $\varepsilon_i^{\bar{\varepsilon}_{-i}^{-i}}$ in the denominator of 5.3, which captures the strategic effect of market expansion on equilibrium prices. Thus, if the prices of A and B are strategic substitutes (i.e. if $\varepsilon_B^{\bar{\varepsilon}_A^A} = \varepsilon_A^{\bar{\varepsilon}_B^B} < 0$ ), then the strategic effect dampens the price change and therefore the competitive part of the total effect on profits.

More importantly, almost symmetry implies that the sign of the total effect on profits is entirely determined by elasticities and independent of the magnitude of the direct effect. In particular, if the new market segment is sufficiently elastic (i.e. if $\varepsilon_A^A = \varepsilon_B^B$ exceeds 1 by a sufficiently large amount), then the total effect is negative and an increase in the magnitude of the (positive) direct effect contributes to exacerbating the negative effect of market expansion on profits. To understand this, suppose $q^A$ becomes larger, i.e. the new market segment is scaled up independently of the existing market and its elasticities. By almost symmetry, the new market segment also increases in relative importance for firm B, so $\frac{q^A}{Q^A}$ and $\frac{q^B}{Q^B}$ increase in parallel by the same amount. Thus, the larger $q^A$ that increases the positive direct effect on A's profits is also associated with a larger $\frac{q^B}{Q^B}$, which makes B's price reduction more aggressive (we continue to assume $\varepsilon_A^A = \varepsilon_B^B > 1$), which in turn hurts A's profits. Thus, if the elasticity of the new market segment is sufficiently large relative to 1, then the increase in $q^A$ actually exacerbates the negative effect on A's profits.

### 5.3 Effect on equilibrium profit ratio

Finally, when the two firms' profits move in the same direction, one may want to know which firm gains or loses more (in relative terms) from market expansion. Thus, we are interested in determining the sign of the effect of market expansion on the ratio between the two profits, i.e. $\frac{d\left(\frac{\pi^A}{\pi^B}\right)}{dM}$. To do so, we use $\pi_i^* = p_i^* Q^i$ and

$$\frac{d\,\pi_i^*}{d\,M} = \frac{\partial\,\pi_i}{\partial\,M} + \frac{\partial\,\pi_i}{\partial\,p_{-i}}\frac{d\,p_{-i}^*}{d\,M} = p_i^* q^i + p_i^* Q_{-i}^i \frac{d\,p_{-i}^*}{d\,M}$$

to obtain

$$\frac{d\left(\frac{\pi_A^*}{\pi_B^*}\right)}{dM}\frac{\pi_B^*}{\pi_A^*} = \frac{\frac{d\pi_A^*}{dM}}{\pi_A^*} - \frac{\frac{d\pi_B^*}{dM}}{\pi_B^*} = \underbrace{\left(\frac{q^A}{Q^A} - \frac{q^B}{Q^B}\right)}_{\text{direct effect}} + \underbrace{\left(\frac{Q_B^A}{Q^A}\frac{dp_B^*}{dM} - \frac{Q_A^B}{Q^B}\frac{dp_A^*}{dM}\right)}_{\text{competitive effect}}$$

Thus, the sign of $\frac{d\left(\frac{\pi_A^*}{\pi_B^*}\right)}{dM}$ is determined by two terms. The first term corresponds to the direct effect of relative market size gains on the profit split. If the relative market size gain for firm A due to market expansion $(\frac{q^A}{Q^A})$ is larger than the relative market size gain for firm B, then the direct effect is positive. This effect is easily understood. The second term captures the difference between the respective percentage market size changes due to changes in the rival firm's price (competitive effect). If market expansion leads firm A to raise its price and firm B to drop its price, then the competitive effect on the ratio between A's profits and B's profits is negative (and vice versa when firm A drops its price and firm B raises its price). If market expansion leads both firms to raise their prices, then the competitive effect on $\frac{\pi^A}{\pi^B}$ is positive whenever the percentage market size (or profit) increase for firm A due to firm B raising its price is larger than the percentage market size (or profit) increase for firm B due to firm A raising its price.

We can then plug in the expressions of $\frac{dp_A}{dM}$ and $\frac{dp_B}{dM}$ from 3.9 to obtain the following corollary.

**Corollary 5.** *The effect of market expansion on the revenue split is*

$$\frac{d\left(\frac{\pi_A^*}{\pi_B^*}\right)}{dM}\frac{\pi_B^*}{\pi_A^*} = \left(\frac{q^A}{Q^A} - \frac{q^B}{Q^B}\right) + \frac{\frac{q^B}{Q^B}\left(1-\varepsilon_B^B\right)\left(\overline{\varepsilon}_B^A\varepsilon_A^{\overline{\varepsilon}_A^A} - \overline{\varepsilon}_A^B\varepsilon_B^{\overline{\varepsilon}_A^A}\right) - \frac{q^A}{Q^A}\left(1-\varepsilon_A^A\right)\left(\overline{\varepsilon}_A^B\varepsilon_B^{\overline{\varepsilon}_B^B} - \overline{\varepsilon}_B^A\varepsilon_A^{\overline{\varepsilon}_B^B}\right)}{\varepsilon_A^{\overline{\varepsilon}_A^A}\varepsilon_B^{\overline{\varepsilon}_B^B} - \varepsilon_A^{\overline{\varepsilon}_B^B}\varepsilon_B^{\overline{\varepsilon}_A^A}}.$$

Let us focus on the competitive effect. For simplicity, assume that the new market segment is less elastic for firm B ($1-\varepsilon_B^B > 0$) and of unit elasticity for firm A. This assumption allows us to focus only on the first part of the competitive effect: $\frac{q^B}{Q^B}\left(1-\varepsilon_B^B\right)\left(\overline{\varepsilon}_B^A\varepsilon_A^{\overline{\varepsilon}_A^A} - \overline{\varepsilon}_A^B\varepsilon_B^{\overline{\varepsilon}_A^A}\right)$. If there was no strategic effect, then A would unambiguously gain (in relative profit terms) from B raising its price: $\frac{q^B}{Q^B}\left(1-\varepsilon_B^B\right)\overline{\varepsilon}_B^A\varepsilon_A^{\overline{\varepsilon}_A^A} > 0$. Things are more complicated when the strategic effect is taken into account . If B raises its price and A views the two prices as strategic complements (i.e. $\varepsilon_B^{\overline{\varepsilon}_A^A} > 0$), then A wants to increase its price, which in turn benefits B. This benefit is larger when B is more substitutable for A (i.e. $\overline{\varepsilon}_A^B$ is larger). Thus, for B to benefit proportionately more than A does $\left(\varepsilon_B^{\overline{\varepsilon}_A^A}/\varepsilon_A^{\overline{\varepsilon}_A^A} > \overline{\varepsilon}_B^A/\overline{\varepsilon}_A^B\right)$, the strategic complementarity $\left(\varepsilon_B^{\overline{\varepsilon}_A^A}\right)$ must be strong enough relative to A's second-order self elasticity$\left(\varepsilon_A^{\overline{\varepsilon}_A^A}\right)$ or consumers must be more willing to substitute from A to B than from B to A.

## 6 Complementors

Suppose now that instead of being competitors, firms A and B are complementors, i.e. are selling complementary rather than substitute products. Formally, the only modification is that $q^i\left(p_i, p_{-i}, m\right)$ is now assumed to be decreasing in $p_{-i}$ , which implies that $Q^i\left(p_i, p_{-i}, m\right)$ is also decreasing in $p_{-i}$ for $i \in \{A, B\}$, so that we now have $\overline{\varepsilon}_B^A < 0$ and $\overline{\varepsilon}_A^B < 0$.

Thus, the entire formal analysis conducted in the previous four sections (including Proposition 1) goes through unchanged. The total effect of market expansion on equilibrium profits can still be decomposed in the same way:

$$\frac{d\,\pi_i^*}{d\,M} = \underbrace{\frac{\partial\,\pi_i}{\partial\,M}}_{\text{direct effect}} + \underbrace{\frac{\partial\,\pi_i}{\partial\,p_{-i}}\frac{d\,p_{-i}^*}{d\,M}}_{\text{complementary effect}} . \tag{6.1}$$

The only difference is that, since $\frac{\partial\,\pi_i}{\partial\,p_{-i}} = p_i \frac{\partial\,Q^i}{\partial\,p_{-i}} < 0$, the sign of the complementary effect on firm $i$'s profits is now the opposite of the sign of $\frac{d\,p_{-i}^*}{d\,M}$. In other words, whenever market expansion leads one firm to increase its price, the complementary effect on the other firm's profits is now negative. On the other hand, the signs of the two components of the complementary effect—non-strategic (myopic) and strategic—of market expansion on equilibrium prices are determined in the same way as before, so Corollary 3 holds unchanged.

## 7 Examples with linear demands

In this section, we provide simple examples with linear demands in order to briefly illustrate some of our main results.

### 7.1 Symmetric competitors

Consider the following symmetric and linear demands

$$q^i\left(p_i, p_{-i}, m\right) = \begin{cases} 1 - \alpha\left(m\right) p_i + \beta\left(m\right)\left(p_{-i} - p_{-i}\right) & \text{if } m \in [0, M_0] \\ \gamma_0\left(1 - \alpha_0 p_i + \beta_0\left(p_{-i} - p_{-i}\right)\right) & \text{if } m \in [M_0, 1] \end{cases}$$

for $i \in \{A, B\}$. All parameters $\left(\alpha\left(m\right), \beta\left(m\right)\right)$ for $m \in [0, M_0]$ and $(\alpha_0, \beta_0, \gamma_0)$ are positive.

Note that $\beta\left(m\right) > 0$ measures the intensity of competition (substitutability) between the two firms within market segment $m$. The interval $[0, M_0]$ represents the existing market segments, whereas $\gamma_0$ measures the size of the new segments. We are interested in the effect on equilibrium profits of adding a segment from$[M_0, 1]$. To do so, we will derive the symmetric equilibrium profits $\pi^*\left(M\right)$ for $M \in [M_0, 1]$, take the derivative in $M$ and evaluate it at $M = M_0$.

Aggregate demands for $M \in [M_0, 1]$ are then

$$Q^i(p_i, p_{-i}, M) = (1 + \gamma_0 (M - M_0)) - (\overline{\alpha} + \gamma_0 \alpha_0 (M - M_0)) p_i + (\overline{\beta} + \gamma_0 \beta_0 (M - M_0)) (p_{-i} - p_i),$$

where we have denoted

$$\overline{\alpha} \equiv \int_0^{M_0} \alpha(m)\, dm \text{ and } \overline{\beta} \equiv \int_0^{M_0} \beta(m)\, dm.$$

The symmetric equilibrium is then easily calculated to be

$$p^*(M) = \frac{1 + \gamma_0 (M - M_0)}{2\overline{\alpha} + \overline{\beta} + \gamma_0 (2\alpha_0 + \beta_0)(M - M_0)}$$

$$\pi^*(M) = \frac{(1 + \gamma_0 (M - M_0))^2 \left(\overline{\alpha} + \overline{\beta} + \gamma_0 (\alpha_0 + \beta_0)(M - M_0)\right)}{\left(2\overline{\alpha} + \overline{\beta} + \gamma_0 (2\alpha_0 + \beta_0)(M - M_0)\right)^2}$$

From here, it is straightforward to obtain

$$\frac{d\pi^*}{dM}|_{M=M_0} = \gamma_0 \frac{\left(\overline{\alpha} + \overline{\beta}\right)\left(2\overline{\alpha} + \overline{\beta}\right) - \left(2\overline{\alpha} + 3\overline{\beta}\right)(\alpha_0 - \overline{\alpha}) - \overline{\beta}\left(\beta_0 - \overline{\beta}\right)}{\left(2\overline{\alpha} + \overline{\beta}\right)^3}.$$

First, note that the sign of the profit change does not depend on $\gamma_0$, the size of the new market segment, consistent with Corollary 4.[8] Second, the sign of the profit change depends on the excess sensitivities of the new market segment in price relative to the average sensitivities in the existing market, i.e. $\alpha_0 - \overline{\alpha}$ and $\beta_0 - \overline{\beta}$. The larger either of these relative sensitivities, the more likely the profit change to be negative. If the new market segment has the same elasticities as the average existing market segments, i.e. if $\alpha_0 = \overline{\alpha}$ and $\beta_0 = \overline{\beta}$, then the profit change is positive as expected. In this case, demands are isomorphic, so adding the additional market segments does not change the equilibrium price ($p^*(M) = \frac{1}{2\overline{\alpha} + \overline{\beta}}$ for all $M$), but profits increase because demand is now larger.

[8] It is worth emphasizing that the derivative must be evaluated at $M = M_0$. If we were to evaluate it at $M > M_0$, then a change in $\gamma_0$ would also change the size of the existing market (not just of the marginal segment), so unsurprisingly, the sign of the profit change would also depend on $\gamma_0$.

## 7.2 Asymmetric complementors

We now use a linear demand example to illustrate the possibility that market expansion into a more price elastic segment for both firms may result in a higher equilibrium price for one of the two firms. However, the linear demand assumption constrains prices to be strategic complements when the two firms are competitors, so the scenario discussed prior to Corollary 3 cannot occur.[9] As a result, we focus on the case in which the two firms are complementors and the linear demands are given by

$$q^i\left(p_i, p_{-i}, m\right) = \begin{cases} 1 - p_i - p_{-i}; & \text{if } m \in [0, M_0] \\ 1 - \beta_i p_i - p_{-i} & \text{if } m \in [M_0, 1] \end{cases}$$

for $i \in \{A, B\}$, where $\beta_i > 1$.

The price equilibrium for $M \in [M_0, 1]$ is easily calculated to be

$$p_i^*\left(M\right) = \frac{2M\left(M_0 + \beta_{-i}\left(M - M_0\right)\right) - M^2}{4\left(M_0 + \beta_i\left(M - M_0\right)\right)\left(M_0 + \beta_{-i}\left(M - M_0\right)\right) - M^2}.$$

The change in price due to the addition of a small segment from $[M_0, 1]$ is then

$$\frac{dp_i^*}{dM}|_{M=M_0} = \frac{2}{9M_0}(1 - 2\beta_i + \beta_{-i}).$$

For example, if $\beta_A = 2$ and $\beta_B = 5$, then $\frac{dp_A^*}{dM}|_{M=M_0} > 0 > \frac{dp_B^*}{dM}|_{M=M_0}$. Thus, even though both firms enter a more elastic market, their prices move in different directions—one firm actually increases its price. However, note that for all $\left(\beta_A, \beta_B\right) \in \left(1, +\infty\right)^2$, at least one of the two prices must decrease, consistent with Corollary 3 (recall that this Corollary also applies to the case when the two firms are complementors).

[9] Recall that A's first order self-elasticity is $-\frac{dQ^A}{dp_A}/\frac{p_A}{Q^A}$. With linear demand, the numerator is a constant. Thus, when B increases its price, $Q^A$ increases (because the two firms are competitors), and therefore A's first order self-elasticity decreases. This incentivizes A to increase its price as well.

## 8 Implications for empirical estimations

As Proposition 1 has shown, correctly estimating the first-order and second-order price elasticities in the existing market is essential for getting the sign of the total effects on prices and profits right. Some commonly used empirical models for estimating demand, while convenient, are not sufficiently flexible for properly estimating these elasticities, which can lead to incorrect predictions, both quantitatively and qualitatively. In Section 7.2 we have already pointed out that linear demand constrains competitors' prices to be strategic complements. The limitations of the Logit model (cf. Lancaster, 1971, McFadden, 1974, 1980) are well known to empirical researchers (Ackerberg et al., 2007). This is why the BLP framework (Berry et al., 1995, 2004) is often adopted in order to accomodate consumer heterogeneity. In this section, we show that the discrete choice model leads to qualitatively different estimations of the effect of market expansion depending on the underlying sources of consumer heterogeneity. We summarize the key building blocks of this example and the results it entails in the remainder of this section, while details are provided in Appendix E.

Consider two firms A and B facing symmetric demands. Following Lancaster (1971), we adopt the discrete choice framework: each consumer buys from A or B (not both), or does not buy anything at all. Consumer $i$'s net utility from buying firm $k$'s product is $u_i(k) = \theta_i^k - \alpha_i p_k + \epsilon_i^k$, where $k = A, B$ and the $\epsilon_i^A$'s and $\epsilon_i^B$'s are independently and identically distributed, following a Logit distribution. Suppose there are only two types of consumers, each making up 50% of the total population, which is normalized to 1. We focus on three separate cases, each with a different source of consumer heterogeneity.

1. *Heterogeneity in intrinsic preferences*: type I consumers have an intrinsic preference for A over B, i.e. $\theta_I^A = \theta^H > \theta^L = \theta_I^B$, whereas type II consumers have an intrinsic preference for B over A, i.e. $\theta_{II}^B = \theta^H > \theta^L = \theta_{II}^A$. Price sensitivities are equal, i.e. $\alpha_I = \alpha_{II} = \alpha$. We vary the demand parameters such that i) each firm's market share in the *aggregate* consumer population is 40% (i.e. 20% of consumers buy nothing), ii) the price equilibrium is $p_A^* = p_B^* = 1$, and iii) each firm's *conditional* market share among the type of consumers by whom it is preferred varies from 40% to 80%.

2. *Heterogeneity in willingness-to-pay*: type I consumers value A and B more highly than type II consumers, i.e. $\theta_I^A = \theta_I^B = \theta^H > \theta^L = \theta_{II}^A = \theta_{II}^B$ . Price sensitivities remain equal, i.e. $\alpha_I = \alpha_{II} = \alpha$. We vary demand parameters such that i) each firm's market share in the *aggregate* consumer population is 40%, ii) the price equilibrium is $p_A^* = p_B^* = 1$, and iii) the conditional market share of each firm among type I consumers varies from 40% (this occurs when $\theta^H = \theta^L$) to 50%.

3. *Heterogeneity in price sensitivity*: type I consumers are more price-sensitive than type II consumers, i.e. $\alpha_I = \alpha^H > \alpha^L = \alpha_{II}$. In this case, we assume $\theta_I^A = \theta_I^B = \theta_{II}^A = \theta_{II}^B = \theta$. Here, we vary demand parameters such that i) each firm's market share in the *aggregate* consumer population is 40%, ii) the price equilibrium is always $p_A^* = p_B^* = 1$, and iii) the ratio of $\alpha_H/\alpha_L$ varies from 1 to 4.

We assume the addressable market for firms A and B expands through the addition of a new market segment, in which the first-order self elasticities at the prevailing prices ($p_A^* = p_B^* = 1$) are $\varepsilon_A^A = \varepsilon_B^B = 2.5$. We are interested in determining the effect of this market expansion for each case.

We have imposed the constraint that each firm's aggregate market share remains fixed at 40% and the prevailing price equilibrium is always $p_A^* = p_B^* = 1$ in order to keep the three models comparable. This also implies that the Logit model will always estimate the effect of market expansion to be the same across all three cases. Indeed, the Logit model assumes that all consumers have the same baseline utility and price sensitivity, i.e. $\theta_i^k = \theta$ and $\alpha_i = \alpha$ for all $i$ and $k$ in our symmetric model, so the elasticities $\bar{\varepsilon}_B^A$, $\varepsilon_B^{\bar{\varepsilon}_B^B}$ and $\varepsilon_B^{\bar{\varepsilon}_A^A}$ are always estimated to have the same value, regardless of the underlying demand structure. As a result, in all three cases, the Logit model predicts that a 1% market expansion results in a 0.29% *decrease* in profit.

Now compare this prediction with the real effect of market expansion for each of the three sources of consumer heterogeneity:

1. *Heterogeneity in intrinsic preferences*: as each firm's conditional market share among the type of consumers by whom it is preferred varies from 40% (no difference in preferences) to 80%, the real effect varies from a 0.29% profit *decrease* to a 1% profit *increase*, as shown in the first panel of Figure 8.1. As consumer preferences are more different (in a horizontal

sense), the competitive effect of market expansion decreases, eventually leading to a positive total effect on profits. At the extreme, preferences are so different that consumers no longer view A and B as substitutes and each of them only considers either A or B. In this case, there is no competitive effect and the conditional market share is 80% (recall that 20% of consumers buy nothing), so a 1% market expansion leads to a 1% increase in profits.

2. *Heterogeneity in willingness-to-pay*: as each firm's conditional market among consumers of the high type (i.e. type I) varies from 40% ($\theta^H = \theta^L$) to 50% , the real effect of a 1% market expansion varies from a 0.29% profit *decrease* to a 0.38% profit *decrease* (see the second panel in Figure 8.1).

3. *Heterogeneity in price sensitivity*: as the ratio of price sensitivities$\alpha_H/\alpha_L$ varies from 1 (everyone is equally price sensitive) to 4, the real effect of a 1% market expansion varies from a 0.29% profit *decrease* to a 2.07% profit *decrease* (see the third panel in Figure 8.1).

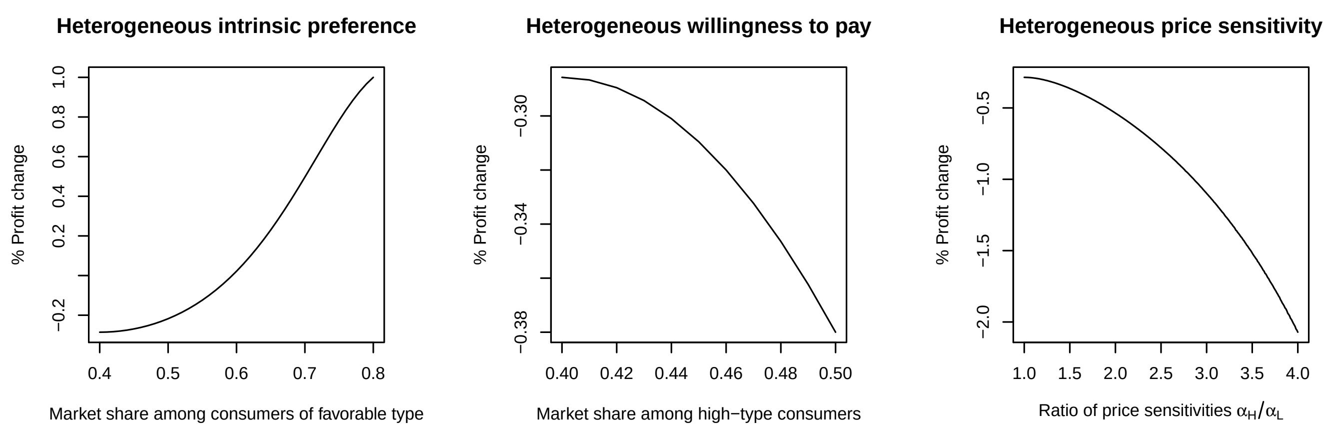


Fig. 8.1: Profit Change for Different Sources of Consumer Heterogeneity

Thus, the exact source of consumer heterogeneity matters: the standard Logit model yield an excessively pessimistic prediction if consumers mainly differ in terms of their favorite product among A and B. However, it yields an excessively optimistic prediction if consumers mainly differ in their willingness to pay for A and B or in their price sensitivity. Now consider a market where consumers mainly differ in their price sensitivity, but are quite similar in terms of their intrinsic preferences and willingness-to-pay. Suppose we used an empirical model that only accomodates differences in intrinsic preferences and/or willingness-to-pay, but not differences in price sensitivity. The prediction of such a model would always be excessively optimistic: if only the first two types of consumer heterogeneity are included, then the worst prediction for a 1% market expansion is a 0.38% decrease in profit, whereas the correct model could yield a profit decrease of more than 2%. Thus, to accurately predict the effect of market expansion, an empirical demand model needs to accomodate the type of consumer heterogeneity that is most relevant to the market studied.

## 9 Conclusion

Our analysis of the effects of exogenous market expansion on a duopoly's equilibrium prices and profits has several clear managerial implications. First and most straightforward, whereas an exogenous expansion of the addressable market is always good news for a monopolist, it can be bad news for firms selling *competing* products in an oligoplistic market—even in the absence of new entry—if the newly added segment is *more* price elastic relative to the average existing segment. Similarly, market expansion can be bad news for firms selling *complementary* products if the newly added segment is *less* price elastic relative to the average existing segment. Second, whether firms' profits increase or decrease as a result of market expansion does not depend on the absolute size of the newly added market segment, but rather on the ratio between the *relative* sizes of the new market segment to the existing segments for the two firms. Third, even if the new market segment is more (respectively, less) price elastic for both firms, it may be optimal for one firm to increase (respectively, decrease) its price if firms' prices are strategic substitutes. This is true both when firms are competitors and when they are complementors. Fourth, we have shown that adopting sufficiently flexible empirical frameworks is crucial to reaching accurate forecasts of the effect of market expansion. However, implementing such empirical frameworks can require prohibitively expensive data-science resources. If such resources are not available, then managers should first determine the main source of differences among their customers, then perform the analysis with a simple Logit model, and finally adjust the results based on the main source of customer heterogeneity based on our analysis in Section 8. For example, if customers differ mainly in terms of their price sensitivity, then the real effect is likely more negative than the predicted effect.

At a more abstract level, we have shown that the mechanism through which market expansion leads to unexpected effects on firm profits can be obtained and explained by relying on traditional substitutability/complementarity and own-price elasticities. In particular, while *strategic* substitutability/complementarity does affect the magnitude of the effects on profits, it is not the crucial factor determining their sign.

There are several promising directions in which our work here can be extended. First and most straightforward, one could apply the same methodology we have employed here to study the impact of exogenous shocks other than market expansion (e.g. uniform demand shifts, new entry) in a similarly general framework. Second, one could introduce network effects in consumer demand and investigate how their magnitude changes the effect of market expansion on firm prices and profits.

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## A Generalizing to non-zero marginal cost

For firm $i \in \{A, B\}$ with marginal cost $c_i$, we redefine "residual demand" as a function of the premium or mark up ($\tilde{p}_i = p_i - c_i$) the firm charges:

$$\tilde{Q}^i(\tilde{p}_i, \tilde{p}_{-i}, M) = Q^i(p_i = \tilde{p}_i + c_i, p_{-i} = \tilde{p}_{-i} + c_{-i}, M).$$

With this transformation, all of our calculations go through, provided demand functions are replaced with residual demand functions, and prices are replaced with markups. Note that all elasticities are now defined in terms of markups and residual demands as well. Thus, even if the original demand function has constant elasticity, the effective elasticity defined in terms of markups and residual demands may not be constant.

## B Proof of Lemma 1

Applying $Q^B_B = -\bar{\varepsilon}^B_B \frac{Q^B}{p^B}$ and $\bar{\varepsilon}^B_B = 1$ at equilibrium, we have

$$\frac{\partial^2 \pi_B}{\partial p_B^2} = Q^B_B + \frac{\partial\left(p_B Q^B_B\right)}{\partial p_B} = Q^B_B - \frac{\partial\left(\bar{\varepsilon}^B_B Q^B\right)}{\partial p_B} = Q^B_B - \frac{\partial \bar{\varepsilon}^B_B}{\partial p_B} Q^B - \bar{\varepsilon}^B_B Q^B_B = -\varepsilon^{\bar{\varepsilon}^B_B}_B \frac{\bar{\varepsilon}^B_B}{p_B} Q^B$$
$$= -\frac{Q^B}{p_B}\varepsilon^{\bar{\varepsilon}^B_B}_B$$

Similarly,

$$\frac{\partial^2 \pi_B}{\partial p_A \partial p_B} = Q^B_A + \frac{\partial\left(p_B Q^B_B\right)}{\partial p_A} = Q^B_A - \frac{\partial\left(\bar{\varepsilon}^B_B Q^B\right)}{\partial p_A} = Q^B_A\left(1 - \bar{\varepsilon}^B_B\right) - Q^B \frac{\partial \bar{\varepsilon}^B_B}{\partial p_A} = -Q^B \frac{\partial \bar{\varepsilon}^B_B}{\partial p_A}$$
$$= \frac{Q^B}{p_A}\varepsilon^{\bar{\varepsilon}^B_B}_A$$

Finally,

$$\frac{\partial^2 \pi_A}{\partial p_A \partial M} = \frac{\partial\left(p_A q^A\right)}{\partial p_A} = q^A + p^A \frac{\partial q^A}{\partial p^A} = q^A - q^A \varepsilon^A_A = q^A(1 - \varepsilon^A_A)$$

## C Proof of Proposition 1

Starting with the expression of $\frac{dp^A}{dM}$ from equation 3.4 and replacing the various terms with their expressions as functions of elasticities from Lemma 1, we obtain:

$$\frac{dp^A}{dM} = \frac{\frac{Q^A}{p^B}\varepsilon^{\bar{\varepsilon}^A_A}_B q^B\left(1 - \varepsilon^B_B\right) + \frac{Q^B}{p^B}\varepsilon^{\bar{\varepsilon}^B_B}_B q^A\left(1 - \varepsilon^A_A\right)}{\frac{Q^A Q^B}{p^A p^B}\left(\varepsilon^{\bar{\varepsilon}^A_A}_A \varepsilon^{\bar{\varepsilon}^B_B}_B - \varepsilon^{\bar{\varepsilon}^B_B}_A \varepsilon^{\bar{\varepsilon}^A_A}_B\right)} = p^A \frac{\frac{q^A}{Q^A}\varepsilon^{\bar{\varepsilon}^B_B}_B\left(1 - \varepsilon^A_A\right) + \frac{q^B}{Q^B}\varepsilon^{\bar{\varepsilon}^A_A}_B\left(1 - \varepsilon^B_B\right)}{\varepsilon^{\bar{\varepsilon}^A_A}_A \varepsilon^{\bar{\varepsilon}^B_B}_B - \varepsilon^{\bar{\varepsilon}^B_B}_A \varepsilon^{\bar{\varepsilon}^A_A}_B}.$$

And similarly for $\frac{dp^B}{dM}$.

We can then write the total effect on profits:

$$\frac{d\,\pi_B}{d\,M} = \frac{\partial\,\pi_B}{\partial\,M} + \frac{\partial\,\pi_B}{\partial\,p_A}\frac{d\,p_A}{d\,M} = p_B q_B + \left(\frac{p^B}{p^A} Q^B \overline{\varepsilon}_A^B\right)\frac{d\,p_A}{d\,M}$$
$$= p_B q_B + \left(\frac{p^B}{p^A} Q^B \overline{\varepsilon}_A^B\right) p^A \frac{\frac{q^A}{Q^A}\varepsilon_B^{\overline{\varepsilon}_B^B}\left(1-\varepsilon_A^A\right) + \frac{q^B}{Q^B}\varepsilon_B^{\overline{\varepsilon}_A^A}\left(1-\varepsilon_B^B\right)}{\varepsilon_A^{\overline{\varepsilon}_A^A}\varepsilon_B^{\overline{\varepsilon}_B^B} - \varepsilon_A^{\overline{\varepsilon}_B^B}\varepsilon_B^{\overline{\varepsilon}_A^A}}$$
$$= p_B q_B \left\{1 + \overline{\varepsilon}_A^B \frac{\frac{q^A}{q^B}\frac{Q^B}{Q^A}\varepsilon_B^{\overline{\varepsilon}_B^B}\left(1-\varepsilon_A^A\right) + \varepsilon_B^{\overline{\varepsilon}_A^A}\left(1-\varepsilon_B^B\right)}{\varepsilon_A^{\overline{\varepsilon}_A^A}\varepsilon_B^{\overline{\varepsilon}_B^B} - \varepsilon_A^{\overline{\varepsilon}_B^B}\varepsilon_B^{\overline{\varepsilon}_A^A}}\right\}.$$

## D Proof of Corollary 3

Assume $\varepsilon_A^A > 1$ and $\varepsilon_B^B > 1$ throughout. If in addition $\varepsilon_B^{\overline{\varepsilon}_A^A} > 0$, then $\frac{d\,p^A}{d\,M} < 0$ (from expression 5.1 on page 16) and, similarly, if $\varepsilon_A^{\overline{\varepsilon}_B^B} > 0$, then $\frac{d\,p^B}{d\,M} < 0$. Suppose therefore that both firms view prices as strategic substitutes, i.e. $\varepsilon_B^{\overline{\varepsilon}_A^A} < 0$ and $\varepsilon_A^{\overline{\varepsilon}_B^B} < 0$. Then

$$sign\left(\frac{d\,p^A}{d\,M}\right) = sign\left(1 + \frac{q^A}{q^B}\frac{Q^B}{Q^A}\frac{1-\varepsilon_A^A}{1-\varepsilon_B^B}\frac{\varepsilon_B^{\overline{\varepsilon}_B^B}}{\varepsilon_B^{\overline{\varepsilon}_A^A}}\right),$$
$$sign\left(\frac{d\,p^B}{d\,M}\right) = sign\left(1 + \frac{q^B}{q^A}\frac{Q^A}{Q^B}\frac{1-\varepsilon_B^B}{1-\varepsilon_A^A}\frac{\varepsilon_A^{\overline{\varepsilon}_A^A}}{\varepsilon_A^{\overline{\varepsilon}_B^B}}\right).$$

However, we have

$$\frac{q^A}{q^B}\frac{Q^B}{Q^A}\frac{1-\varepsilon_A^A}{1-\varepsilon_B^B}\frac{\varepsilon_B^{\overline{\varepsilon}_B^B}}{\varepsilon_B^{\overline{\varepsilon}_A^A}} \cdot \frac{q^B}{q^A}\frac{Q^A}{Q^B}\frac{1-\varepsilon_B^B}{1-\varepsilon_A^A}\frac{\varepsilon_A^{\overline{\varepsilon}_A^A}}{\varepsilon_A^{\overline{\varepsilon}_B^B}} = \frac{\varepsilon_A^{\overline{\varepsilon}_A^A}\varepsilon_B^{\overline{\varepsilon}_B^B}}{\varepsilon_A^{\overline{\varepsilon}_B^B}\varepsilon_B^{\overline{\varepsilon}_A^A}} > 1,$$

where the last inequality is implied by the stability condition 3.8 on page 10. Thus, since both $\frac{q^A}{q^B}\frac{Q^B}{Q^A}\frac{1-\varepsilon_A^A}{1-\varepsilon_B^B}\frac{\varepsilon_B^{\overline{\varepsilon}_B^B}}{\varepsilon_B^{\overline{\varepsilon}_A^A}}$ and $\frac{q^B}{q^A}\frac{Q^A}{Q^B}\frac{1-\varepsilon_B^B}{1-\varepsilon_A^A}\frac{\varepsilon_A^{\overline{\varepsilon}_A^A}}{\varepsilon_A^{\overline{\varepsilon}_B^B}}$ are negative, we conclude that at least one of them must be smaller than -1, so $\frac{d\,p^A}{d\,M} < 0$ or $\frac{d\,p^B}{d\,M} < 0$.

## E Details of the numerical example used in section 8

Recall that consumer $i$'s net utility from buying firm $k$'s product is $u_i(k) = \theta_i^k - \alpha_i p_k + \epsilon_i^k$, where $k = A, B$ and the $\epsilon_i^A$'s and $\epsilon_i^B$'s are independently and identically distributed, following a Logit distribution. There are two types of consumers, each making up 50% of the total population, which is normalized to 1. The *conditional* market shares of the two firms within the population of type I consumers are

$$s_I^A = \frac{\exp(\theta_I^A - \alpha_I p_A)}{1+\exp(\theta_I^A - \alpha_I p_A)+\exp(\theta_I^B - \alpha_I p_B)}$$
$$s_I^B = \frac{\exp(\theta_I^B - \alpha_I p_B)}{1+\exp(\theta_I^A - \alpha_I p_A)+\exp(\theta_I^B - \alpha_I p_B)}.$$

And symmetrically for the conditional market shares of the two firms, $s_{II}^A$ and $s_{II}^B$, within the population of type II consumers. The *aggregate* market shares in the total consumer population are then $\bar{s}^A = \frac{1}{2}\left(s_I^A + s_{II}^A\right)$ and $\bar{s}^B = \frac{1}{2}\left(s_I^B + s_{II}^B\right)$.

In this setting, given that equilibrium prices are $p_A^* = p_B^* = 1$, we can derive the first and second-order elasticities as functions of the conditional market shares realized in equilibrium:[10]

$$\bar{\varepsilon}_B^B = \frac{E\left[\alpha s^B(1-s^B)\right]p_B}{E\left[s^B\right]} = \frac{1}{s_I^B + s_{II}^B}\left[\alpha_I s_I^B\left(1-s_I^B\right)+\alpha_{II} s_{II}^B\left(1-s_{II}^B\right)\right] \tag{E.1}$$

$$\bar{\varepsilon}_A^B = \frac{E\left[\alpha s^B s^A\right]p_A}{E\left[s^B\right]}. \tag{E.2}$$

$$\varepsilon_B^{\bar{\varepsilon}_B^B} = 2 - \frac{E\left[\alpha^2\left(1-2s^B\right)s^B(1-s^B)\right]}{E\left[s^B\right]}. \tag{E.3}$$

$$\varepsilon_A^{\bar{\varepsilon}_B^B} = \frac{-E\left[\alpha s^B s^A\right] + E\left[\alpha^2\left(1-2s^B\right)s^B s^A\right]}{E\left[s^B\right]}. \tag{E.4}$$

Now consider each of the three dimensions of heterogeneity in turn. In the model with heterogeneous intrinsic preferences, $\theta_I^A = \theta_{II}^B = \theta^H > \theta^L = \theta_I^B = \theta_{II}^A$ and $\alpha_I = \alpha_{II} = \alpha$. Recalling that in equilibrium the first-order conditions in price implies $\bar{\varepsilon}_A^A = \bar{\varepsilon}_B^B = 1$, we can use the expression of $\bar{\varepsilon}_B^B$ from equation (E.1) to determine the value of $\alpha$ corresponding to a given set of conditional

[10] To derive the expressions of $\varepsilon_B^{\bar{\varepsilon}_B^B}$ and $\varepsilon_A^{\bar{\varepsilon}_B^B}$, we also use that $\bar{\varepsilon}_B^B = 1$ at the equilibrium prices, so $E\left[\alpha s^B(1-s^B)\right] = E\left[s^B\right]$.

market shares $\left(s_I^A = s_{II}^B, s_{II}^A = s_I^B = 0.8 - s_I^A\right)$. We then use the value of $\alpha$ to determine the other elasticities from (E.2), (E.3) and (E.4). Finally, we obtain the resulting profit change from expression (5.4). Applying this procedure as $s_I^A$ varies from 40% to 80%, we obtain the first panel in Figure 8.1.

The methodology for the model with heterogeneity in willingness-to-pay is identical. The only difference is that now$\theta_I^A = \theta_I^B = \theta^H > \theta^L = \theta_{II}^B = \theta_{II}^A$, so $s_I^A = s_I^B$ and $s_{II}^A = s_{II}^B = 0.8 - s_I^A$. Here, we vary $s_I^A$ from 40% to 50%.

In the model with heterogeneity in price sensitivities, we have $\theta_I^A = \theta_I^B = \theta_{II}^A = \theta_{II}^B = \theta$ and $\alpha_I = \alpha^H > \alpha^L = \alpha_{II}$. Here, the procedure runs in the following two steps. First, for each $k \equiv \frac{\alpha^H}{\alpha^L}$, the system of equations

$$\begin{cases} \frac{1}{0.8}\left[k\alpha^L s_I^B\left(1 - s_I^B\right) + \alpha^L s_{II}^B\left(1 - s_{II}^B\right)\right] = 1 \\ \frac{\exp(\theta - k\alpha^L)}{1 + 2\exp(\theta - k\alpha^L)} = s_I^B \\ \frac{\exp(\theta - \alpha^L)}{1 + 2\exp(\theta - \alpha^L)} = s_{II}^B \\ s_I^B + s_{II}^B = 0.8 \end{cases}$$

admits a unique solution in $\left(\alpha^L, \theta, s_I^B, s_{II}^B\right)$. Second, for each $k$, we use the $\left(\alpha^L, \theta, s_I^B, s_{II}^B\right)$ determined in the frst step to calculate the remaining elasticities according to expressions (E.2), (E.3) and (E.4). Then use expression (5.4) to derive the resulting profit change.

Finally, consider the Logit model, which ignores the heterogeneity of consumer types and therefore implicitly assumes that the conditional market shares are always equal to the aggregate market share (recall the latter is kept equal to 40%). The elasticities estimated by the Logit model at equilibrium prices are therefore given by

$$\begin{aligned} \bar{\varepsilon}_B^B &= \alpha\left(1 - \bar{s}^B\right) \\ \bar{\varepsilon}_A^B &= \alpha\bar{s}^A \\ \varepsilon_B^{\bar{\varepsilon}_B^B} &= 2 - \alpha^2\left(1 - 2\bar{s}^B\right)\left(1 - \bar{s}^B\right) \\ \varepsilon_A^{\bar{\varepsilon}_B^B} &= \alpha\bar{s}^A\left(-1 + \alpha\left(1 - 2\bar{s}^B\right)\right). \end{aligned}$$

From $\bar{\varepsilon}_B^B = 1$, the Logit model estimates $\alpha = \frac{1}{1-\bar{s}^B}$, which can then be used to determine $\bar{\varepsilon}_A^B$, $\varepsilon_B^{\bar{\varepsilon}_B^B}$ and $\varepsilon_A^{\bar{\varepsilon}_B^B}$. Using expression (5.4), this leads to an estimated profit decrease of 0.29% for a 1% market expansion. Note that this estimate is independent of the real conditional market shares.